

# D-branes in non-tachyonic 0B orientifolds

E. Dudas and J. Mourad

*Laboratoire de Physique Théorique <sup>1</sup>,  
Université de Paris-Sud, Bât. 210, F-91405 Orsay Cedex, France*

## Abstract

We determine all D-branes in the non-tachyonic 0'B orientifold, examine their world-volume anomalies and study orbifold compactifications. We find that the spectrum of the D-branes contains chiral fermions in the symmetric, antisymmetric and fundamental representations of (unitary) gauge groups on the branes. The cancellation of the world-volume anomalies requires Wess-Zumino terms which we determine explicitly. We examine a non-tachyonic compactification to 9D whose closed part interpolates between 0B and IIB and revisit compactifications on orbifolds. The D3-brane allows to conjecture, via the AdS/CFT correspondence, a supergravity dual to a non-supersymmetric and infrared-free gauge theory. The D-string gives hints concerning the S-dual of the 0'B orientifold.

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<sup>1</sup>Laboratoire associé au CNRS-URA-D0063.

# 1 Introduction

D-branes played a major role in understanding nonperturbative properties of string and field theories in the last few years. In addition to their crucial role in string dualities [1], D-branes are also at the heart of the AdS/CFT correspondence [2], relating gauge field theories to string (SUGRA) theories and allowing, in principle, a nonperturbative formulation of supersymmetric string theories. Some non-supersymmetric string theories seem to pass all the consistency tests which characterise the supersymmetric ones like world-sheet superconformal and modular invariance and the absence of space-time anomalies. Most of these theories have tachyons in the perturbative spectrum but there exist (at least) three theories which are non-supersymmetric and with no tachyons: the  $SO(16) \times SO(16)$  heterotic string, the  $USp(32)$  type I string which is an orientifold of type IIB and finally the 0'B string, an orientifold of type 0B with a gauge group  $U(32)$ . The latter has the same Ramond-Ramond (RR) forms as type IIB. So one would expect to have D-branes charged with respect to these forms. On the other hand, D-branes have a conformal field theory definition as open strings with Dirichlet boundary conditions for a set of coordinates [1]. The subject of this paper is to show the existence of these extended objects in 0'B and determine their properties. Unlike their supersymmetric cousins, these branes are not BPS, however the conservation of the RR charge guarantees their stability.

Originally conjectured to relate superconformal field theories on D3 branes to Type IIB string compactifications, the AdS/CFT correspondence was more recently conjectured to apply also to non-conformal and non-supersymmetric theories coming from D3 branes on Type OB [3], [4], [5] or their orientifolds [6], [7], [8], [9]. The non-conformal behaviour of gauge theories was related to the classical background of Type OB and to the closed string tachyon condensation [4] and, for the compactifications of the non-tachyonic orientifold introduced in [6], to the dilaton tadpole of the model [9]. It was argued that the gauge theory on D3 branes is free in the ultraviolet (UV) and that the renormalization group running of the gauge couplings is qualitatively in agreement with the dilaton (and tachyon, whenever it exists) background on the gravity side.

The purpose of the present paper is twofold. First, we analyze in detail the spectrum and the anomaly cancellation for all branes (D7, D5, D3 and D1) present in the non-tachyonic OB orientifold<sup>2</sup>[6]. The consistency conditions we impose are that the world-sheet parity operator squares to one on physical states and that the open string amplitudes have a correct interpretation in the transverse channel as a closed string tree level exchange. The solution to these conditions is conveniently determined by introducing an action of the spacetime and world-sheet fermions numbers on the Chan-Paton matrices. We find that the D-branes have a  $U(D)$  gauge group with chiral fermions in the fundamental, symmetric and

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<sup>2</sup>Properties of branes in Type OB were studied in [10].

antisymmetric representations. These fermions generate world-volume anomalies whose structure is highly constrained by the ten-dimensional Green-Schwarz mechanism. The cancellation of the anomalies allows to verify the consistency of the world-volume theory and to predict new Wess-Zumino couplings on the branes.

Our second purpose is to complete the study of non-tachyonic compactifications to various dimensions. In particular, the simplest non-tachyonic compactification is to 9d, where the massless spectrum of the model displays an interesting combination of Type I compactifications with supersymmetry breaking on the branes (called “brane supersymmetry breaking” in the literature) and in the bulk (called “M-theory breaking” or “brane supersymmetry”). We also study orbifold compactifications down to 8d, 6d and 4d. Tadpole cancellation in these compactifications requires the existence of D-branes, thus allowing an independent check on the world-volume content of the branes. The spectrum of D3 branes corresponds to a gauge theory which is actually strongly coupled in the UV and free in the infrared (IR). We work out the classical background and compare with the perturbative properties of the gauge theory. By working out the spectrum of the D1 brane, we conjecture the existence of a corresponding (non-conventional) fundamental string theory.

The paper is organized as follows. We start with some notations and conventions in Section 2. In Section 3 we define a convenient formalism for finding the spectrum of non-tachyonic OB orientifolds and apply it to the 10d model introduced in [6]. Based on this formalism we study in Section 4 the spectrum of all D branes and study the gauge and gravitational anomaly cancellation. We show how the gauge part of the Wess-Zumino terms can be obtained in a unified manner for all D-branes. In Section 5 we consider non-tachyonic compactifications. First, we study the simplest example in 9D whose closed part interpolates between 0B and IIB. Second, we study orbifolds on  $T^{2n}/Z_2$ . We rederive the spectrum of D7, D5 and D3 branes based on the computation of one-loop orbifold amplitudes in compact space and taking at the end the 10d limit of infinite compact volume. In Section 6 we work out the classical background of the D3-D9 system and show agreement with the massless spectrum on the D3 branes, via the AdS/CFT correspondence. Section 7 summarises our results and conclusions.

## 2 Notations and conventions

Orientifolds [11, 12, 13] are the appropriate context for a perturbative study of D branes and their interactions. In addition to the torus amplitude  $\mathcal{T}$ , at one-loop there are three additional amplitudes to consider, the Klein bottle  $\mathcal{K}$ , the cylinder  $\mathcal{A}$  and the Möbius  $\mathcal{M}$ . It is often convenient for a spacetime particle

interpretation to write the partition functions with the help of  $SO(2n)$  characters

$$\begin{aligned} O_{2n} &= \frac{1}{2\eta^n}(\theta_3^n + \theta_4^n) , & V_{2n} &= \frac{1}{2\eta^n}(\theta_3^n - \theta_4^n) , \\ S_{2n} &= \frac{1}{2\eta^n}(\theta_2^n + i^n \theta_1^n) , & C_{2n} &= \frac{1}{2\eta^n}(\theta_2^n - i^n \theta_1^n) , \end{aligned} \quad (1)$$

where the  $\theta_i$  are the four Jacobi theta-functions with (half)integer characteristics. In a spacetime interpretation, at the lowest level  $O_{2n}$  represents a scalar,  $V_{2n}$  represents a vector, while  $S_{2n}$ ,  $C_{2n}$  represent spinors of opposite chiralities. In order to link the direct and transverse channels, one needs the transformation matrices  $S$  and  $P$  for the level-one  $SO(2n)$  characters (1). These may be simply deduced from the corresponding transformation properties of the Jacobi theta functions, and are

$$S_{(2n)} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & i^{-n} & -i^{-n} \\ 1 & -1 & -i^{-n} & i^{-n} \end{pmatrix} , \quad P_{(2n)} = \begin{pmatrix} c & s & 0 & 0 \\ s & -c & 0 & 0 \\ 0 & 0 & \zeta c & i\zeta s \\ 0 & 0 & i\zeta s & \zeta c \end{pmatrix} , \quad (2)$$

where  $c = \cos(n\pi/4)$ ,  $s = \sin(n\pi/4)$  and  $\zeta = e^{-in\pi/4}$  [13]. The modulus parameter  $q = \exp(2i\pi\tau)$  for the three one-loop surfaces is given by

$$\text{Klein} : \tau = 2i\tau_2 , \quad \text{Annulus} : \tau = \frac{it}{2} , \quad \text{Mobius} : \tau = \frac{it}{2} + \frac{1}{2} , \quad (3)$$

where  $\tau_2 = \text{Im}\tau$  is the imaginary part of the closed string torus amplitude and  $t$  is the open string modulus, analog of the Schwinger proper time in field theory.

In  $d$  noncompact dimensions and  $10 - d$  compact dimensions, the relevant string amplitudes have the symbolic form

$$\begin{aligned} \mathcal{T} &= \frac{1}{(4\pi^2\alpha')^{1+d/2}} \int \frac{d^2\tau}{\tau_2^{1+d/2}} \frac{1}{|\eta(\tau)|^{2d}} T , \quad \mathcal{K} = \frac{1}{(4\pi^2\alpha')^{1+d/2}} \int \frac{dt}{t^{1+d/2}} \frac{1}{\eta(\tau)^d} K , \\ \mathcal{A} &= \frac{1}{(8\pi^2\alpha')^{1+d/2}} \int \frac{dt}{t^{1+d/2}} \frac{1}{\eta(\tau)^d} A , \quad \mathcal{M} = \frac{1}{(8\pi^2\alpha')^{1+d/2}} \int \frac{dt}{t^{1+d/2}} \frac{1}{\eta(\tau)^d} M , \end{aligned} \quad (4)$$

where  $\alpha'$  is the string tension and we explicitly displayed the contribution of the spacetime bosons. The “amputated” amplitudes  $T, K, A, M$  in (4), used for brevity in the rest of the paper, define the remaining part of the amplitudes, which contain the contribution of spacetime fermions, of  $10 - d$  bosons and fermions and, in the compact space case, the compactification lattice. We use a similar notation for Dp-branes in the noncompact case, in which case  $d$  is replaced by the dimension of the brane world-volume,  $p + 1$ .

These amplitudes have the dual interpretation of tree-level closed string exchanges between D branes and O planes. The corresponding closed string modulus,  $l$ , is related to the moduli defined in (3) by

$$\text{Klein} : l = \frac{1}{2\tau_2} , \text{ Annulus} : l = \frac{2}{t} , \text{ Mobius} : l = \frac{1}{2t} . \quad (5)$$

Written in the transverse channel, the amplitudes (4) become

$$\begin{aligned} \mathcal{K} &= \frac{1}{(4\pi^2\alpha')^{1+d/2}} \int dl \frac{1}{\eta(il)^d} \tilde{K} , \\ \mathcal{A} &= \frac{1}{(8\pi^2\alpha')^{1+d/2}} \int dl \frac{1}{\eta(il)^d} \tilde{A} , \mathcal{M} = \frac{1}{(8\pi^2\alpha')^{1+d/2}} \int dl \frac{1}{\eta(il+1/2)^d} \tilde{M} , \end{aligned} \quad (6)$$

where  $\tilde{K}, \tilde{A} (\tilde{M})$  are related by the  $S (P)$  transformation defined in (2) to the loop amplitudes  $K, A (M)$ .

### 3 Non-tachyonic orientifold of 0B

The closed sector of the 0'B orientifold is given by the 0B string [15]. The RNS world-sheet action of the 0B theory is the same as that of the superstring with a different GSO projection, which is given by

$$P_{0B} = \frac{1 + (-1)^{F_R+F_L}}{2}, \quad (7)$$

in the Ramond-Ramond (RR) and Neveu-Schwarz-Neveu-Schwarz (NSNS) sectors. Here  $F_{L,R}$  are the world-sheet fermion numbers. The absence of the RNS sector leads to a space-time theory without fermions. The presence of the  $NS^-NS^-$  sector (with  $(-1)^{F_L} = (-1)^{F_R} = -1$ ) introduces a tachyon in the theory. The  $NS^+NS^+$  sector gives the  $NSNS$  spectrum of IIB and the  $RR$  sector doubles the RR spectrum of IIB. The spectrum and the modular invariance are summarised in the torus partition function

$$T = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2. \quad (8)$$

Sagnotti's 0'B orientifold [6] is obtained by first gauging the discrete symmetry

$$\Omega' = \Omega(-1)^{F_L}, \quad (9)$$

where  $\Omega$  is the usual parity operator on the world-sheet. Notice that

$$\Omega'^2 = (-1)^{F_L+F_R}, \quad (10)$$

which is 1 due to the GSO projection (7). The tachyon is not invariant under  $\Omega'$  and so is removed from the spectrum. In the  $NS^+NS^+$  and the  $R^+R^+$  sectors

$\Omega'$  acts as  $\Omega$  and so the two-form is removed from the gravitational multiplet, as well as the zero and the four-form from the first subset of RR forms. Finally from the second subset of RR forms  $\Omega'$  acts as  $-\Omega$  and so the two-form is removed and the axion and the self-dual four form are retained. This is summarised in the Klein amplitude

$$K = \frac{1}{2}(-O_8 + V_8 - S_8 + C_8), \quad (11)$$

where the notation was explained in Section 1. The transverse channel Klein amplitude

$$\tilde{K} = -2^5 S_8, \quad (12)$$

reveals a RR tadpole for the ten-form which has to be cancelled by the introduction of open strings. From the spacetime point of view the self-dual four-form leads to gravitational anomalies whose cancellation (at least their irreducible part) asks for chiral fermions. The open string states carry Chan-Paton degrees of freedom represented by matrices  $\lambda$ . Two open string states may join to form a closed string state, the amplitude being proportional to  $tr(\lambda_1 \lambda_2)$ . Since the closed part has no fermions, one is tempted to look for an open spectrum without fermions. However this cannot be consistent because of spacetime anomalies from the closed spectrum. So one is forced to introduce fermions in the open sector. The absence of fermions in the closed sector and their presence in the open sector can be reconciled by the decomposition of the set of Chan-Paton matrices,  $\mathcal{M}$ , as a  $Z_2$  graded algebra  $\mathcal{M} = \mathcal{M}^+ \oplus \mathcal{M}^-$ . Recall that this means that an element of  $\mathcal{M}$  is either an element of  $\mathcal{M}^+$  (a boson) or an element of  $\mathcal{M}^-$  (a fermion) and that the product respects the graduation  $\mathcal{M}^+ \times \mathcal{M}^+ \in \mathcal{M}^+$ ,  $\mathcal{M}^- \times \mathcal{M}^- \in \mathcal{M}^+$  and  $\mathcal{M}^+ \times \mathcal{M}^- \in \mathcal{M}^-$ . Let the fermions have Chan-Paton matrices in  $\mathcal{M}^-$  and the bosons in  $\mathcal{M}^+$ . Then  $(-1)^G$ , where  $G$  is the spacetime fermion number, has an action on the Chan-Paton factors

$$(-1)^{G_{cp}}(\lambda) = \sigma_9 \lambda \sigma_9^{-1}, \quad (13)$$

with  $\sigma_9$  a unitary matrix which verifies<sup>3</sup>  $\sigma_9^2 = 1$ . The Chan-Paton matrices of fermions verify

$$(-1)^{G_{cp}}(\lambda_f) = -\lambda_f, \quad (14)$$

and those of bosons

$$(-1)^{G_{cp}}(\lambda_b) = \lambda_b, \quad (15)$$

that is the action of  $(-1)^G$  on the whole state (oscillators,  $G_o$ , and Chan-Paton matrices,  $G_{cp}$ ) gives 1 for both fermions and bosons. This resolves the problem of the joining of one fermionic and one bosonic open string into a closed one. In fact the amplitude is proportional to

$$tr(\lambda_f \lambda_b) = tr(\sigma_9 \lambda_f \sigma_9^{-1} \sigma_9 \lambda_b \sigma_9^{-1}) = -tr(\lambda_f \lambda_b) = 0. \quad (16)$$

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<sup>3</sup>There is phase choice here, that we choose to be equal to one.

Let the action of  $\Omega'$  on the Chan-Paton matrices be given by  $\gamma_9$ ,

$$\Omega'(\lambda) = \gamma_9 \lambda^T \gamma_9^{-1} . \quad (17)$$

Then since one has  $\Omega^2 = 1$  on the oscillators,  $\gamma_9$  has to satisfy

$$\gamma_9^T = \epsilon \gamma_9 , \quad (18)$$

where  $\epsilon = \pm 1$ . Another constraint on  $\gamma_9$  comes from  $(-1)^G \Omega' = \Omega' (-1)^G$ ,

$$\sigma_9^T = \eta \gamma_9^{-1} \sigma_9 \gamma_9 , \quad (19)$$

where  $\eta = \pm 1$ . More constraints are obtained from the requirement of the tadpole cancellation. The cylinder amplitude is given by

$$A = \frac{1}{2} \text{Tr} \left\{ (-1)^{G_o} \frac{1 + (-1)^F}{2} \frac{1 + (-1)^{G_o + G_{cp}}}{2} e^{-\pi t L_0} \right\} , \quad (20)$$

where the trace is on the open sector modes. The result is

$$A = \frac{1}{4} \left\{ [(tr 1)^2 + (tr \sigma_9)^2] V_8 - [(tr 1)^2 - (tr \sigma_9)^2] S_8 \right\} . \quad (21)$$

The arguments of the characters were defined in Section 1. Here we have used the GSO projection on the open sector  $(-1)^F = 1$ , a more general setting in the next section will clarify this choice. Here, we note that this GSO projection is compatible with the single valuedness of the operator product expansion as well as with the absence of non diagonal sectors in the closed sector.

The Mobius amplitude is given by

$$M = \frac{1}{2} \text{Tr} \left\{ \Omega (-1)^{G_o} \frac{1 + (-1)^F}{2} \frac{1 + (-1)^{G_o + G_{cp}}}{2} e^{-\pi t L_0} \right\} , \quad (22)$$

where the trace is on the open sector modes. We find

$$M = -\frac{1}{4} \epsilon [(1 + \eta)(tr 1) V_8 - (1 - \eta)(tr 1) S_8] . \quad (23)$$

From (21) and (23) we deduce the transverse channel amplitudes

$$\begin{aligned} \tilde{A} &= \frac{2^{-5}}{4} [(tr 1)^2 (V_8 - S_8) + (tr \sigma_9)^2 (O_8 - C_8)] , \\ \tilde{M} &= -\frac{1}{2} \epsilon [(1 + \eta)(tr 1) V_8 - (1 - \eta)(tr 1) S_8] . \end{aligned} \quad (24)$$

The RR tadpole cancellation gives the constraints

$$tr \sigma_9 = 0 \quad , \quad 2^{-5} (tr 1)^2 - 2\epsilon (1 - \eta)(tr 1) + 4 \times 2^5 = 0 . \quad (25)$$

The first constraint together with  $\sigma_9^2 = 1$  allows  $\sigma_9$  to be chosen such that  $\sigma_9 = \text{diag}(1_{N \times N}, -1_{N \times N}) = 1 \otimes \sigma_3$ . The second constraint implies that  $\epsilon = 1, \eta = -1$  and  $N = 32$ . The first two conditions can be solved by  $\gamma_9 = 1 \otimes \sigma_1$ . This gives the gauge bosons with Chan-Paton factors  $\lambda_v = \text{diag}(h, -h^T)$ , with  $h$  a hermitian  $32 \times 32$  matrix and so the D9 gauge group is  $U(32)$ . The Chan-Paton matrices of the Weyl fermions are of the form

$$\lambda_f = \begin{pmatrix} 0 & A \\ -A^* & 0 \end{pmatrix}, \quad (26)$$

with  $A$  an antisymmetric matrix. The fermions thus belong to the antisymmetric representation of  $U(32)$ . Note that the tadpole from the NSNS sector is non vanishing. It induces a term in the effective action from the disk diagram

$$\Delta S_{\text{tadpole}} = -32T_9 \int \sqrt{-g} e^{-\phi}, \quad (27)$$

where  $T_9$  is the tension of the D9 branes. The consistency of the model from the spacetime point of view can be verified by calculating the total anomaly due to the fermions from the open sector and the self-dual form from the closed sector. The anomaly polynomial<sup>4</sup> has the form [6]

$$X_{12} = X_2 X_{10} + X_4 X_8 + \frac{1}{2} X_6^2, \quad (28)$$

with

$$\begin{aligned} X_2 &= i \text{tr} F, \quad X_4 = -\frac{1}{2} (\text{tr} F^2 - \text{tr} R^2), \\ X_6 &= -\frac{i}{6} \left( \text{tr} F^3 - \frac{1}{8} \text{tr} F \text{tr} R^2 \right), \\ X_8 &= \frac{1}{24} \left( \text{tr} F^4 + \frac{1}{8} \text{tr} R^4 + \frac{1}{32} (\text{tr} R^2)^2 - \frac{1}{8} \text{tr} F^2 \text{tr} R^2 \right), \\ X_{10} &= \frac{i}{48} \left( \frac{2}{5} \text{tr} F^5 + \frac{1}{240} \text{tr} F \text{tr} R^4 - \frac{1}{192} \text{tr} F (\text{tr} R^2)^2 \right). \end{aligned} \quad (29)$$

The irreducible anomaly in  $\text{tr} R^6$  has cancelled and this particular factorised form allows a Green-Schwarz mechanism to cancel the remaining anomaly. The Green-Schwarz terms in the effective action read

$$\Delta S_{GS} = T_{-1} \int_{D9} A_0 X_{10} + T_1 \int_{D9} A_2 X_8 + \frac{T_3}{2} \int_{D9} A_4 X_6, \quad (30)$$

where  $A_0, A_2$  and  $A_4$  are respectively the RR axion, two-form and self dual four-form. We shall see in the next section that  $T_i$  is the tension of a  $i$ -brane. The

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<sup>4</sup>Here the actual value of the anomaly polynomial is  $\hat{X}_{12} = 2\pi X_{12}$ . We shall equally use this convention in the rest of the paper.



Green-Schwarz mechanism is completed by the modification of the Bianchi identity of these forms

$$dF_{i+1} = \frac{2\pi}{T_{i-1}} X_{i+2} , \quad i = 0, 2, 4 , \quad (31)$$

so that the variation of the forms under gauge transformations are given by

$$\delta A_i = -\frac{2\pi}{T_{i-1}} X_i^1 , \quad i = 0, 2, 4 , \quad (32)$$

where the  $X_i^1$  are related to  $X_{i+2}$  by the descent equations

$$X_{i+2} = dX_{i+1}, \quad \delta X_{i+1} = dX_i^1 . \quad (33)$$

The field equations of the RR forms are modified due to (30)

$$d^* F_{i+1} = 2T_{i-1} \kappa^2 X_{10-i} , \quad i = 0, 2, 4 , \quad (34)$$

where  $\kappa^2$  is the ten-dimensional gravitational constant. Notice that the factor  $1/2$  in (30) in front of the term in  $A_4$  is essential in obtaining the field equation (34) for the self-dual form<sup>5</sup>. The self-duality of  $F_5$  implies that  $T_3$  has a fixed value

$$T_3^2 = \frac{\pi}{\kappa^2} , \quad (35)$$

a result which we shall derive from a string calculation of the amplitude between two branes in the next section. Note that the Dirac quantization condition (with minimum charge)  $T_i T_{6-i} \kappa^2 = \pi$  allows to write both the field equations and the Bianchi identities as (31) with the index  $i$  taking even values from 0 to 8. We shall show in the next section how each polynomial  $X_{i+2}$  gives the irreducible world-volume anomaly of the  $p = i-1$  D-brane, providing a remarkable relation between space-time and world-volume anomaly cancellation. The basic idea is that the variations (32) induce a non-trivial transformation of the Wess-Zumino term of the  $D$  brane  $T_i \int_D A_{i+1}$  which has to be cancelled by the world-volume anomalies. This is similar to the relation between the usual Green-Schwarz mechanism and the world-volume anomalies of the string [19] and the five-brane [20, 21, 22, 23].

## 4 0'B D-branes

From the RR forms of 0'B we expect to have  $D(2p+1)$  branes for  $p = -1, 0, 1, 2, 3, 4$ . The 9-branes were considered in the previous section. Here we determine from the world-volume point of view the consistent theories on the branes. There are essentially two consistency conditions: first  $\Omega$  should square to one on the Dirichlet-Dirichlet (DD) and the Neumann-Dirichlet (ND) states and second the

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<sup>5</sup>This is easily verified by going to 9 dimensions, where the self-dual form gives one three-form, or more covariantly by using the PST formalism [18].

cylinder and Mobius amplitudes should have a correct interpretation in the transverse channel as an exchange of physical closed string states. We shall confirm the massless excitations from the world-volume anomaly analysis in this section as well as from the explicit computation of the string amplitudes in orbifold compactifications in the next section.

The excitations of the D-branes come from two sectors: the DD sector where the open string has both ends on the D-brane and the ND sector with one end on the brane and the other on the 9-brane. As before, there are bosonic and fermionic Chan-Paton factors distinguished by the value of  $(-1)^{G_{cp}}$ , which is 1 in the NS sector and  $-1$  in the  $R$  sector. We denote the corresponding matrix by  $\sigma_p$ . Similarly, to the world-sheet fermion number  $(-1)^F$  we introduce an action on the Chan-Paton factors

$$(-1)^{F_{cp}}(\lambda) = f_p \lambda f_p^{-1} , \quad (36)$$

where  $f_p$  is a unitary matrix which squares to one and commutes with  $\sigma_p$ . The presence of this action on the Chan-Paton factors is needed in order to insure that two open strings join to form a closed string with only diagonal sectors.

The action of  $\Omega$  on the oscillator degrees of freedom in the DD sector is the same as the action in the NN sector except that there is in addition a space-time parity operation on the transverse space to the brane, that can be written symbolically as

$$\Omega_{DD} = \pi \Omega_{NN} . \quad (37)$$

When acting in the  $R$  sector, the zero mode part of the parity is given by  $\pi_0 = \Pi_i(\Gamma\Gamma^i)$ , where  $i$  labels the  $9 - p$  transverse coordinates and  $\Gamma$  is the ten dimensional chirality matrix. The square of  $\pi_0$  is given by

$$\pi_0^2 = (-1)^{(9-p)(10-p)/2} , \quad (38)$$

which is  $+1$  for  $p = 4n + 1, 4n + 2$  and  $-1$  for  $p = 4n, 4n + 3$ . For the total action of  $\Omega^2$  to be one it is necessary that the action of  $\Omega^2$  on the fermionic Chan-Paton factors be given by the value of  $\pi_0^2$ . Therefore, if the action of  $\Omega$  on the DD Chan-Paton matrices is given by  $\gamma_p$ , we have

$$\gamma_p(\gamma_p^{-1})^T = \epsilon_p , \quad (39)$$

for  $p = 4n + 1, 4n + 2$  and

$$\gamma_p(\gamma_p^{-1})^T = \epsilon'_p \sigma_p \quad (40)$$

for  $p = 4n, 4n + 3$ . Equation (39) implies that  $\epsilon_p = \pm 1$  and equation (40) may be rewritten as

$$\gamma_p = (\epsilon'_p)^2 \sigma_p \gamma_p \sigma_p^T , \quad (41)$$

which, using  $\sigma_p^2 = 1$  gives  $(\epsilon'_p)^4 = 1$ . The values of  $\epsilon_p$  and  $\epsilon'_p$  are determined from the ND sector. Here also we have to impose that the square of  $\Omega$  be 1 on the

tensor product of the oscillator part and the Chan-Paton part of the states. The action of  $\Omega^2$  on the oscillators can be deduced, following [14], by considering the product of a ND vertex operator and a DN vertex operator. In the R sector, where the supercurrent  $T_F$  is periodic,  $\Omega^2$  acts as  $i^{(9-p)/2}$  and in the NS sector, where  $T_F$  is antiperiodic,  $\Omega^2$  acts as  $i^{(p-1)/2}$ . On the other hand, the Chan-Paton factors in the Ramond ND sector satisfy

$$\sigma_9 \lambda \sigma_p^{-1} = -\lambda. \quad (42)$$

The Neveu-Schwarz ND Chan-Paton factors obey a similar relation except for a plus sign on the right hand side. The action of  $\Omega^2$  on the Ramond ND Chan-Paton factors must satisfy

$$\gamma_9 (\gamma_9^{-1})^T \lambda \gamma_p^T \gamma_p^{-1} = (i)^{(p-9)/2} \lambda \quad (43)$$

and in the NS sector they must satisfy

$$\gamma_9 (\gamma_9^{-1})^T \lambda \gamma_p^T \gamma_p^{-1} = (-i)^{(p-9)/2} \lambda. \quad (44)$$

The tadpole cancellation gave  $\gamma_9 (\gamma_9^{-1})^T = 1$ . Consider the case  $p = 4n + 1$ , then using (39), equations (43) and (44) become

$$\lambda = \epsilon_p (-1)^n \lambda, \quad (45)$$

which gives  $\epsilon_1 = 1, \epsilon_5 = -1$  for the the D string and the D5-brane. Now consider the case  $p = 4n + 3$ , then (40) allows (43) and (44) to be written as

$$\lambda \sigma_p^{-1} = i \epsilon'_p (-1)^n \sigma_9 \lambda \sigma_p^{-1}, \quad (46)$$

where we used the action of  $(-1)^G$  on the Chan-Paton factors. Multiplying (46) by  $\sigma_p$  leads to  $(1 - i(-1)^n \epsilon'_p \sigma_p) \lambda = 0$ . This is consistent provided  $\epsilon'_p = \pm i$ . Finally we get  $\epsilon'_3 = \pm i$  and  $\epsilon'_7 = \pm i$ . The commutation of  $(-1)^G$  and  $(-1)^F$  with  $\Omega$  gives

$$\sigma_p^T = \eta_p \gamma_p^{-1} \sigma_p \gamma_p, \quad f_p^T = \eta'_p \gamma_p^{-1} f_p \gamma_p, \quad (47)$$

whose compatibility with (41) for  $p = 3$  and  $p = 7$  gives

$$\eta_p = \epsilon_p'^2 = -1, \quad p = 3, 7. \quad (48)$$

So far the constraints were obtained from the absence of non-diagonal closed strings and the existence of states where  $\Omega$  squares to one on the tensor product of Chan-Paton and oscillator degrees of freedom. There are other constraints which come from the requirement that the open string diagrams (the cylinder and the Mobius) have a dual interpretation. Indeed, after a modular transformation the diagrams have the interpretation of tree level closed strings diagrams. Consider the cylinder amplitude for DD open strings

$$A_{p-p} = \frac{1}{2} \text{Tr} \left\{ (-1)^{G_o} \frac{1 + (-1)^{F_o + F_{cp}}}{2} \frac{1 + (-1)^{G_o + G_{cp}}}{2} e^{-\pi t L_0} \right\}, \quad (49)$$

where the trace is on the DD open string states. It reads

$$\begin{aligned}
A_{p-p} = & \frac{1}{8} \left\{ [(tr1)^2 + (tr\sigma_p)^2 + (trf_p)^2 + (trf_p\sigma_p)^2]V_8 \right. \\
& + [(tr1)^2 + (tr\sigma_p)^2 - (trf_p)^2 - (trf_p\sigma_p)^2]O_8 \\
& - [(tr1)^2 - (tr\sigma_p)^2 + (trf_p)^2 - (trf_p\sigma_p)^2]S_8 \\
& \left. - [(tr1)^2 - (tr\sigma_p)^2 - (trf_p)^2 + (trf_p\sigma_p)^2]C_8 \right\} \frac{1}{\eta^{9-p}} . \quad (50)
\end{aligned}$$

A modular transformation brings the amplitude to the form

$$\tilde{A}_{p-p} = \frac{2^{-(p+1)/2}}{4} l^{\frac{p-9}{2}} \left[ (tr1)^2 V_8 + (tr\sigma_p)^2 O_8 - (trf_p)^2 S_8 - (trf_p\sigma_p)^2 C_8 \right] \frac{1}{\eta^{9-p}} . \quad (51)$$

The character  $O_8$  has the interpretation of the exchange between the branes of closed string states whose lowest mass is the tachyon. The latter has been removed by the  $\Omega'$  projection so this term in the transverse channel amplitude has to be absent. This is possible if

$$tr\sigma_p = 0 , \quad (52)$$

which is another consistency requirement on  $\sigma_p$ . By a suitable change of basis  $\sigma_p$  may be put in the form  $\sigma_p = 1_{M \times M} \otimes \sigma_3$ . The one and five brane couple to RR fields that originate from the  $|S_8|^2$  sector of 0B, whereas the 3 and 7 brane couple to the RR forms that come from the  $|C_8|^2$  sector, so we get the constraints

$$tr(f_p\sigma_p) = 0 \quad , \quad p = 1, 5 , \quad (53)$$

$$tr(f_p) = 0 \quad , \quad p = 3, 7 . \quad (54)$$

The Mobius amplitude also has to have a correct transverse channel interpretation as an exchange of closed string states between the  $O9$  and the D-brane. The direct channel amplitude reads

$$\begin{aligned}
M_p = & -\frac{1}{8}(1 + \eta_p)tr(\gamma_p^{-1}\gamma_p^T) \left[ (1 + \eta'_p)(V_{p-1}O_{9-p} - O_{p-1}V_{9-p}) \right. \\
& + (1 - \eta'_p)(O_{p-1}O_{9-p} - V_{p-1}V_{9-p}) \left. \right] \frac{1}{\eta^{9-p}} \\
& + \frac{1}{8}i^{(9-p)(10-p)/2}(1 - \eta_p)tr(\gamma_p^{-1}\gamma_p^T) \left[ (1 + \eta'_p)(S_{p-1}S_{9-p} - C_{p-1}C_{9-p}) \right. \\
& + (1 - \eta'_p)(C_{p-1}S_{9-p} - S_{p-1}C_{9-p}) \left. \right] \frac{1}{\eta^{9-p}} , \quad (55)
\end{aligned}$$

and a  $P$  transformation gives the transverse amplitude

$$\tilde{M}_p = -\frac{1}{4}(1 + \eta_p)tr(\gamma_p^{-1}\gamma_p^T) \left[ (1 + \eta'_p) \left( \cos\left(\frac{p-1}{2}\pi\right)(V_{p-1}O_{9-p} - O_{p-1}V_{9-p}) \right. \right.$$

$$\begin{aligned}
& - \sin\left(\frac{p-1}{2}\pi\right)O_8) - (1 - \eta'_p)(O_{p-1}O_{9-p} - V_{p-1}V_{9-p})\left]\frac{1}{\eta^{9-p}}\right. \\
& + \frac{1}{4}(1 - \eta_p)\text{tr}(\gamma_p^{-1}\gamma_p^T)\left[(1 + \eta'_p)\left(\cos\left(\frac{p-1}{2}\pi\right)(S_{p-1}S_{9-p} - C_{p-1}C_{9-p})\right.\right. \\
& + \left.\left.i\sin\left(\frac{p-1}{2}\pi\right)(C_{p-1}S_{9-p} - S_{p-1}C_{9-p})\right)\right. \\
& + \left.\left.(1 - \eta'_p)\left(\cos\left(\frac{p-1}{2}\pi\right)(C_{p-1}S_{9-p} - S_{p-1}C_{9-p})\right.\right.\right. \\
& + \left.\left.\left.i\sin\left(\frac{p-1}{2}\pi\right)(S_{p-1}S_{9-p} - C_{p-1}C_{9-p})\right)\right]\frac{1}{\eta^{9-p}}. \tag{56}
\end{aligned}$$

The  $O9$  plane couples only to the RR fields originating from the  $R^+R^+$  sector, so the Mobius amplitude in the transverse channel must only contain the contribution of  $S_8$ . On the other hand the D3 and D7 branes couple to  $R^-R^-$  fields, so the Mobius amplitude must vanish for these branes and indeed this is the case due to (40) and (52). For the other branes, (the D1 and D5 branes), we get the additional constraints

$$1 + \eta_p = 0, \quad 1 - \eta'_p = 0, \quad p = 1, 5. \tag{57}$$

The first insures that the Mobius amplitude does not contain an exchange of NSNS fields and the second that it contains no  $R^-R^-$  modes. Notice that (48) and (57) imply that  $\eta_p = -1, \forall p$ .

It is consistent to ask for the absence of open string tachyons for states stretched between two branes. In fact, the conditions

$$f_p = 1, \quad p = 1, 5 \tag{58}$$

$$f_p = \sigma_p, \quad p = 3, 7, \tag{59}$$

insure that the amplitudes (50) do not involve the open string tachyon. In this case the cylinder amplitudes take the simple form

$$A_{p-p} = \frac{(tr1)^2}{4} (V_8 - S_8), \quad p = 1, 5, \tag{60}$$

$$A_{p-p} = \frac{(tr1)^2}{4} (V_8 - C_8), \quad p = 3, 7. \tag{61}$$

It remains to consider the interaction of the D-branes with the 9-branes filling the spacetime. This is described by the cylinder amplitude between the 9 and the D brane. It is given in the direct channel by

$$\begin{aligned}
A_{9-p} &= \frac{1}{4} \left\{ [(tr1_9)(tr1_p) + tr(\sigma_9)tr(\sigma_p) + tr(f_9)tr(f_p) + tr(\sigma_9 f_9)tr(\sigma_p f_p)] \right. \\
&\quad (V_{p-1}S_{9-p} + O_{p-1}C_{9-p}) \\
&\quad + \left. [(tr1_9)(tr1_p) + tr(\sigma_9)tr(\sigma_p) - tr(f_9)tr(f_p) - tr(\sigma_9 f_9)tr(\sigma_p f_p)] \right\}
\end{aligned}$$

$$\begin{aligned}
& (O_{p-1}S_{9-p} + V_{p-1}C_{9-p}) \\
& - [(tr1_9)(tr1_p) - tr(\sigma_9)tr(\sigma_p) + tr(f_9)tr(f_p) - tr(\sigma_9f_9)tr(\sigma_pf_p)] \\
& (S_{p-1}O_{9-p} + C_{p-1}V_{9-p}) \\
& - [(tr1_9)(tr1_p) - tr(\sigma_9)tr(\sigma_p) - tr(f_9)tr(f_p) + tr(\sigma_9f_9)tr(\sigma_pf_p)] \\
& (C_{p-1}O_{9-p} + S_{p-1}V_{9-p}) \left\} \left( \frac{\eta}{\theta_4} \right)^{(9-p)/2}.
\end{aligned} \tag{62}$$

Here we have used the relations  $(-1)^{F_o} = (-1)^{F_o^{(p-1)}} (-1)^{F_o^{(9-p)}}$  and the convention that  $O_{p-1}$  (resp.  $V_{p-1}$ ) corresponds to  $(-1)^{F_o^{(p-1)}} = -1$  (resp.  $1$ ), while  $O_{9-p}$  (resp.  $V_{9-p}$ ) corresponds to  $(-1)^{F_o^{(9-p)}} = 1$  (resp.  $-1$ ). Similarly,  $S$  and  $C$  correspond respectively to the  $+1$  and  $-1$  values of  $(-1)^{F_o}$ . We have also used the fact that each antiperiodic boson contributes with  $\sqrt{\eta/\theta_4}$  to the partition function. After using (52, 53, 54) and (58, 59) as well as  $f_9 = 1$  and  $tr\sigma_9 = 0$ , (62) simplifies to

$$\begin{aligned}
A_{9-p} &= \frac{(tr1_9)(tr1_p)}{2} [V_{p-1}S_{9-p} + O_{p-1}C_{9-p} - S_{p-1}O_{9-p} - C_{p-1}V_{9-p}] \\
&\times \left( \frac{\eta}{\theta_4} \right)^{\frac{9-p}{2}}, \text{ for } p = 1, 5 \\
A_{9-p} &= \frac{(tr1_9)(tr1_p)}{4} \left\{ (V_{p-1} + O_{p-1})(S_{9-p} + C_{9-p}) \right. \\
&\left. - (S_{p-1} + C_{p-1})(V_{9-p} + O_{9-p}) \right\} \left( \frac{\eta}{\theta_4} \right)^{\frac{9-p}{2}}, \text{ for } p = 3, 7.
\end{aligned} \tag{63}$$

The transverse amplitudes read

$$\begin{aligned}
\tilde{A}_{9-p} &= \frac{2^{-(p+1)/2}}{2} (tr1_9)(tr1_p) \left\{ V_{p-1}O_{9-p} - O_{p-1}V_{9-p} \right. \\
&\left. - (-1)^{(9-p)/4} (S_{p-1}S_{9-p} - C_{p-1}C_{9-p}) \right\} \left( \frac{\eta}{\theta_2} \right)^{\frac{9-p}{2}}
\end{aligned} \tag{64}$$

for  $p = 1, 5$  and

$$\tilde{A}_{9-p} = \frac{2^{-(p+1)/2}}{4} (tr1_9)(tr1_p) [V_{p-1}O_{9-p} - O_{p-1}V_{9-p}] \left( \frac{\eta}{\theta_2} \right)^{\frac{9-p}{2}} \tag{65}$$

for  $p = 3, 7$ , which indeed represent the exchange of physical closed string modes between the Dp and the D9 brane.

Notice that (60,61) imply that the cylinder amplitude between two identical branes is numerically zero. That is, there is a boson-fermion degeneracy in the open sector and a corresponding cancellation between the NSNS and RR closed string exchange. On the other hand, the interaction between the 9-brane and the D-brane, given by  $\tilde{A}_{9-p}$  is non-vanishing except for  $p = 5$ . Let  $N_0(p)$  be the

minimum rank of the matrices representing the Chan-Paton matrices. Then the tension of the branes can be deduced from the transverse amplitude (60,61) to be

$$T_p^{0'B} = T_p \frac{N_0(p)}{2} , \quad (66)$$

where  $T_p$  is the corresponding tension in type  $IIB$  given by

$$T_p^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{3-p} . \quad (67)$$

## 4.1 D3-brane

Since we have chosen  $\sigma_3$  to be of the form

$$\sigma_3 = \begin{pmatrix} 1_{M \times M} & 0 \\ 0 & -1_{M \times M} \end{pmatrix} , \quad (68)$$

the bosonic Chan-Paton matrices are diagonal

$$\lambda_b = \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix} , \quad (69)$$

and the fermionic ones off-diagonal

$$\lambda_f = \begin{pmatrix} 0 & Y_1 \\ Y_2 & 0 \end{pmatrix} . \quad (70)$$

In the DD sector,  $X_1$  and  $X_2$  are hermitian and  $Y_1^\dagger = Y_2$ . They are further constrained by the invariance under  $\Omega$ . Recall that  $\gamma_3$  anticommutes with  $\sigma_3$  and verifies  $\gamma_3(\gamma_3^{-1})^T = i\sigma_3$ <sup>6</sup>. A solution is given by  $\gamma_3 = 1_{M \times M} \otimes \left( \frac{\sigma_1 + \sigma_2}{\sqrt{2}} \right)$ :

$$\gamma_3 = \begin{pmatrix} 0_{M \times M} & e^{i\pi/4} 1_{M \times M} \\ e^{-i\pi/4} 1_{M \times M} & 0_{M \times M} \end{pmatrix} . \quad (71)$$

The DD Chan-Paton matrices for the vectors obey  $\lambda_v = -\gamma_3 \lambda_v^T \gamma_3^{-1}$ , with  $\lambda_v$  of the form (69). This is solved by  $\lambda_v = \text{diag}(h, -h^T)$  where  $h$  are hermitian  $M \times M$  matrices. The resulting gauge group is thus  $U(M)$ . There are also 6 massless scalars representing the fluctuations of the brane. Their Chan-Paton matrices verify  $\lambda_s = \gamma_3 \lambda_s^T \gamma_3^{-1}$  and so have the form  $\lambda_s = \text{diag}(h, h^T)$ . They are in the adjoint of  $U(M)$ . The GSO projection in the R sector is onto states with  $(-1)^F = -1$ . The states are characterised by their four dimensional chirality  $\epsilon$  which is correlated with the six-dimensional transverse one  $\epsilon' = -\epsilon$ . The massless fermions with four-dimensional chirality  $\epsilon$  have Chan-Paton factors of the form (70), verifying  $\lambda_{f\epsilon} = i\epsilon \gamma_3 \lambda_{f\epsilon}^T \gamma_3^{-1}$ . This is due to the fact that  $\Omega$  acts as  $i\epsilon'$  on the

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<sup>6</sup>The other choice  $\gamma_3(\gamma_3^{-1})^T = -i\sigma_3$  leads to the same spectrum with a global chirality change.

oscillator part of the state. We get negative chirality fermions with Chan-Paton matrices of the form

$$\lambda_{f-} = \begin{pmatrix} 0 & S \\ S^* & 0 \end{pmatrix}, \quad (72)$$

with  $S$  a symmetric complex matrix and for positive chirality fermions

$$\lambda_{f+} = \begin{pmatrix} 0 & A \\ -A^* & 0 \end{pmatrix}, \quad (73)$$

with  $A$  an antisymmetric matrix. The fermionic spectrum from the DD sector comprises thus, one fermion in the  $(2_-, 4_+, M(M+1)/2)$  and one in the  $(2_+, 4_-, M(M-1)/2)$  of  $SO(1,3) \times SO(6) \times U(M)$ . This completes the massless spectrum of the DD excitations. Note that it differs from the spectrum proposed in [8]. The ND sector gives massless states with Chan-Paton matrices  $\lambda$  constrained by the condition  $\Omega^2 = 1$

$$\lambda\sigma_3 = \pm\lambda, \quad (74)$$

where the plus sign is for the Ramond sector and the minus sign for the Neveu-Schwarz sector. Since  $F_{cp} = \sigma_3$  the GSO projection is onto  $(-1)^{F_0} = 1$  for the R sector and  $(-1)^{F_0} = -1$  for the NS sector. From the NS sector we get only massive states. The massless states from the R sector have negative chirality and Chan-Paton factors of the form

$$\begin{pmatrix} 0 & 0 \\ m & 0 \end{pmatrix}, \quad (75)$$

with  $m$  a complex  $32 \times M$  matrix. The corresponding states transform in the  $(\bar{32}, \bar{M})$  of  $U(32) \times U(M)$  and are singlets under the transverse  $SO(6)$  Lorentz group. In the DN Ramond sector Chan-Paton factors must obey  $\sigma_3\lambda = -\lambda$  and so the GSO projection is onto  $(-1)^{F_0} = -1$ . We get a massless positive chirality fermion with a Chan-Paton matrix

$$\lambda = \begin{pmatrix} 0 & 0 \\ 0 & m \end{pmatrix}, \quad (76)$$

that transform in the  $(32, M)$  of  $U(32) \times U(M)$ . Notice that the ND fermions have two degrees of freedom as well as the DN fermions. Their invariant combination under  $\Omega$  gives one physical positive chirality fermion in the  $(32, M)$  of  $U(32) \times U(M)$ .

The smallest value  $M = 1$  gives  $N_0(3) = 2$  and thus the tension of the 3-branes is given from (66) by  $T_3^2 = \pi/(\kappa^2)$ , in accord with (35).

Now we are in position to calculate the anomaly polynomial of the D3-brane<sup>7</sup>. The potential anomalies are that of the transformations of  $U(M) \times U(32) \times$

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<sup>7</sup>This computation is very similar to the one performed for the Type I D5 brane in [22].



$SO(1, 3) \times SO(6)$ . We will denote by  $G$  (resp.  $F$ ) the field strength of the  $U(M)$  (resp.  $U(32)$ ) gauge bosons, by  $R$  the induced space-time curvature on the world-volume and finally by  $N$  the curvature of the normal group  $SO(6)$ . If  $\Sigma$  denotes the world-volume curvature, then  $tr R^2 = tr \Sigma^2 + tr N^2$ . We shall use as well the relations

$$\begin{aligned} Tr_A^S G &= (M \pm 1)tr G, \quad Tr_A^S G^2 = (M \pm 2)tr G^2 + (tr G)^2, \\ Tr_A^S G^3 &= (M \pm 4)tr G^3 + 3tr(G)tr(G^2), \end{aligned} \quad (77)$$

where  $Tr_A^S$  is the trace in the antisymmetric or symmetric representation of  $U(M)$  and  $tr$  is as usual the trace in the fundamental representation. Using the standard anomaly formulas, as found for example in [16], [17], the anomaly of the world-volume fermions of the D3-brane reads

$$\begin{aligned} I_6 &= -i \left[ M \frac{1}{6} \left( tr F^3 - \frac{1}{8} tr R^2 tr F \right) + \frac{1}{2} tr F \left( \frac{M}{24} tr N^2 + tr G^2 \right) \right. \\ &\quad \left. + \frac{1}{2} tr G \left( tr F^2 - tr R^2 \right) \right]. \end{aligned} \quad (78)$$

Notice that the irreducible part of the  $U(M)$  anomaly has cancelled. Furthermore the anomaly can be put in the particular form

$$I_6 = MX_6 + Y_2 X_4 + Y_4 X_2, \quad (79)$$

where

$$Y_2 = itr G, \quad Y_4 = -\frac{1}{2} \left( \frac{M}{24} tr N^2 + tr G^2 \right), \quad (80)$$

and  $X_i$  are the polynomials entering in the spacetime anomaly (28). From eqs. (32) we see that (79) has the required form to be cancelled by the following part of the world-volume action

$$MT_3 \int_{D3} A_4 + T_1 \int_{D3} A_2 Y_2 + T_{-1} \int_{D3} A_0 Y_4. \quad (81)$$

The first term is the expected RR coupling of the D3-brane, the two other terms are predictions for the world-volume action. The second is a theta term for the world-volume gauge bosons and the first gives mass to the  $U(1)$  part of the  $U(M)$  gauge fields.

## 4.2 D7-brane

Since  $\gamma_7(\gamma_7^{-1})^T = -i\sigma_3$ , the treatment of the D7-brane is somewhat analogous to the D-3 brane, in particular  $\gamma_7$  is given by the complex conjugate of equation (71). The vectors are  $U(M)$  gauge bosons and there are 2 massless scalars in the adjoint of  $U(M)$ . A difference with respect to the D3-brane arises from the Ramond DD sector. If  $\epsilon$  and  $\epsilon'$  denote the chiralities of the fermions in eight

and two dimensions, then the GSO projection being onto  $(-)^{F_0} = -1$ , we have  $\epsilon = -\epsilon'$ , but  $\Omega$  now acts as  $-i\epsilon'$ , with a sign difference compared to the D3-brane. So the 77 fermions belong to  $(8_+, 1_-, M(M-1)/2)$  and  $(8_-, 1_+, M(M+1)/2)$  of  $SO(1, 7) \times SO(2) \times U(M)$ . From the ND sector the constraint  $\Omega^2 = 1$  gives from (46) and  $\epsilon'_7 = -1$

$$\lambda\sigma_3 = \pm\lambda, \quad (82)$$

where the minus sign is for the Neveu-Schwarz sector and the plus for the Ramond sector. As for the D3-brane, this results in a positive chirality fermion in the  $(32, M)$  of  $U(32) \times U(M)$ . The 97 scalars are however now tachyons transforming in the  $(32, \bar{M})$ .

The anomaly cancellation mechanism is more subtle than the case of the D3-brane because the contribution of the normal bundle is more involved. The anomaly polynomial is given by the ten-form part of

$$I_{10} = \hat{A}(\Sigma)[e^{\frac{N}{2}} Tr_A(e^{iG}) - e^{-\frac{N}{2}} Tr_S(e^{iG}) + tr(e^{iF})tr(e^{iG})], \quad (83)$$

where  $N$  is the  $U(1)$  normal curvature. After some arrangements, explained in the Appendix, it reads

$$\begin{aligned} I_{10} = & M \left( X_{10} + \frac{NY_8}{2} \right) + \left( X_8 + \frac{Y_6N}{2} \right) Y_2 + \left( X_6 + \frac{Y_4N}{2} \right) Y_4 \\ & + \left( X_4 + \frac{Y_2N}{2} \right) Y_6 + \left( X_2 + \frac{MN}{2} \right) Y_8, \end{aligned} \quad (84)$$

where we defined the functions (see the Appendix for more details)

$$\begin{aligned} Y_2 &= itrG, \quad Y_4 = -\frac{1}{2}trG^2 + \frac{M}{48}trR^2 - \frac{M}{48}trN^2, \\ Y_6 &= -i \left( \frac{1}{6}trG^3 - \frac{1}{96}trGtrR^2 \right) - \frac{i}{48}trGtrN^2, \\ Y_8 &= \frac{1}{24} \left( trG^4 + \frac{M}{480}trR^4 - \frac{M}{384}(trR^2)^2 + \frac{1}{4}trG^2trN^2 + \frac{M}{320}(trN^2)^2 \right) \end{aligned} \quad (85)$$

where the trace in the fundamental representation of  $SO(2)$  is  $trN^2 = -2N^2$ . The residual anomaly is cancelled by the world-volume terms in the action

$$MT_7 \int_{D7} A_8 + T_5 \int_{D7} A_6 Y_2 + T_3 \int_{D7} A_4 Y_4 + T_1 \int_{D7} A_2 Y_6 + T_{-1} \int_{D7} A_0 Y_8. \quad (86)$$

These terms modify the equations of motion and the Bianchi identities of the forms in (31) and (34) by  $\delta$  terms localised on the brane

$$dF_{i+1} = \frac{2\pi}{T_{i-1}} (X_{i+2} + \frac{1}{2}Y_i\delta), \quad (87)$$

where  $\delta$  here is a two-form. This results, when combined with the fact that the restriction of  $\delta$  on the world-volume of the brane has a non-vanishing term [32]

$$\delta|_{D7} = N , \quad (88)$$

to a variation of (86) which compensates exactly the anomaly (84). Notice that for the D3-brane the restriction of  $\delta$  term to the world-volume vanishes, which explains the difference in the factorised structure of the anomaly.

### 4.3 D5-brane

For the D5-brane we have  $\gamma_5^T = -\gamma_5$  and  $\{\gamma_5, \sigma_5\} = 0$ , which are solved by  $\gamma_5 = 1_{M \times M} \otimes \sigma_2$ . The Chan-Paton matrices of the DD vector bosons are of the form  $\text{diag}(h, -h^T)$ , with  $h$  a self-adjoint matrix. The gauge group is thus  $U(M)$  and acts with  $g = \text{diag}(g_1, g_1^*)$ . The scalars representing the transverse fluctuations of the brane belong as usual to the adjoint of  $U(M)$ . The GSO projection in the Ramond DD sector is now onto  $(-1)^{F_o} = +1$  and therefore the transverse ( $\epsilon'$ ) and longitudinal ( $\epsilon$ ) chiralities of the fermions are equal. The action of  $\Omega$  on the oscillator part of the fermions is given by  $\epsilon'$ , so the Chan-Paton matrices of the fermions must obey  $\lambda_{f\epsilon} = \epsilon \gamma_5 \lambda_{f\epsilon}^T \gamma_5^{-1}$ . This is solved by

$$\lambda_{f-} = \begin{pmatrix} 0 & S \\ S^* & 0 \end{pmatrix} , \quad (89)$$

where  $S$  is a symmetric  $M \times M$  matrix and

$$\lambda_{f+} = \begin{pmatrix} 0 & A \\ -A^* & 0 \end{pmatrix} , \quad (90)$$

where  $A$  is antisymmetric. The DD fermions are thus in the  $(4_-, 2_-, M(M+1)/2)$  and the  $(4_+, 2_+, M(M-1)/2)$  of  $SO(1, 5) \times SO(4) \times U(M)$ . In the ND sector, the GSO projection is onto  $(-1)^{F_o} = 1$  states. From the Ramond sector, we get one Weyl fermion of positive chirality with Chan-Paton matrix

$$\begin{pmatrix} 0 & \lambda_1 \\ \lambda_2 & 0 \end{pmatrix} , \quad (91)$$

where  $\lambda_1, \lambda_2$  are complex  $32 \times M$  matrices. The corresponding states transform in the bifundamental representation of  $U(32) \times U(M)$ . From the NS sector we get 2 complex massless scalars in the  $(\bar{32}, M)$ .

The anomaly polynomial of the world-volume chiral fermions is given by

$$I_8 = M(X_8 + \frac{Y_4 \chi(N)}{2}) + (X_6 + \frac{Y_2 \chi(N)}{2})Y_2 + (X_4 + \frac{M \chi(N)}{2})Y_4 + X_2 Y_6 , \quad (92)$$

where  $\chi(N)$  is the Euler class of the normal bundle and, as explained in detail in the Appendix, we have defined the functions

$$\begin{aligned} Y_2 &= itrG, \quad Y_4 = \frac{M}{96}trR^2 - \frac{1}{2}trG^2 - \frac{M}{48}trN^2, \\ Y_6 &= -\frac{i}{6}trG^3 - \frac{i}{48}trGtrN^2. \end{aligned} \quad (93)$$

Notice the analogy between (92) and (84). The anomalies are cancelled by the world-volume contribution

$$MT_5 \int_{D5} A_6 + T_3 \int_{D5} A_4 Y_2 + T_1 \int_{D5} A_2 Y_4 + T_{-1} \int_{D5} A_0 Y_6. \quad (94)$$

Similarly to the case of the D7 branes, eq. (87), these terms modify the Bianchi identities. The modified Bianchi identities, combined with the fact the restriction of  $\delta$  on the D5 brane world-volume gives [32]

$$\delta|_{D5} = \chi(N), \quad (95)$$

leads to a variation of (94) which exactly compensates the anomaly (92).

#### 4.4 D string

For the D string it is more convenient to work in a covariant gauge. The D string is characterised by a symmetric  $\gamma_1$  which anticommutes with  $\sigma_1$ :  $\gamma_1 = 1_{M \times M} \otimes \sigma_1$ . The two-dimensional gauge boson  $G$  Chan-Paton matrices have the form  $\lambda = \text{diag}(h, -h^T)$ , giving a  $U(M)$  gauge group which acts with  $g = \text{diag}(g_1, g_1^*)$ . There are also 8 scalars in the adjoint of  $U(M)$ , they represent the transverse fluctuations of the string. The GSO projection in the Ramond DD sector is onto  $(-1)^{F_0} = 1$ , the longitudinal two-dimensional and transverse eight dimensional chiralities are equal. The action of  $\Omega$  on the oscillator part is given by  $-\epsilon'$  and so the Chan-Paton matrices must obey  $\lambda_{f\epsilon} = -\epsilon \gamma_1 \lambda_{f\epsilon}^T \gamma_1^{-1}$ . The positive chirality fermions are in the  $8_+$  of the transverse Lorentz group and have now Chan-Paton factors of the form

$$\lambda_+ = \begin{pmatrix} 0 & A \\ -A^* & 0 \end{pmatrix}, \quad (96)$$

with  $A$  an antisymmetric matrix, while negative chirality fermions transform in the symmetric representation and belong to the  $8_-$  of  $SO(8)$ . In the ND sector, the GSO projection is onto  $(-1)^{F_0} = 1$ . The ND sector gives positive chirality fermions which are singlets under the transverse  $SO(8)$  and with matrices belonging to the  $(\bar{3}2, \bar{M})$  of  $U(32) \times U(M)$ . The anomaly polynomial is given by

$$I_4 = \frac{M}{2}(trR^2 - trF^2) - trGtrF = MX_4 + X_2Y_2, \quad (97)$$

with

$$Y_2 = i \text{tr} G . \quad (98)$$

Notice the absence of terms dependent on the normal curvature. The anomaly is cancelled by

$$T_1 \int_{D_1} A_2 + T_{-1} \int_{D_1} A_0 Y_2 . \quad (99)$$

A particularly interesting case is for one D string. Here the gauge group is  $U(1)$ , the spectrum of the D string comprises in addition to the vector boson, the scalars  $X^i$ , neutral under  $U(1)$ , negative chirality Weyl fermions  $\theta^\alpha$ , with charge 2 under the  $U(1)$  and finally 32 positive chirality Weyl fermions  $\lambda^a$  with charge  $-1$  under the  $U(1)$  and in the fundamental representation of  $U(32)$ . The index  $\alpha$  is a spinorial index of  $SO(8)$ . The world sheet fermions from the DD sector look like the Green-Schwarz fermions of the superstring except that they are now complex. The fermions  $\lambda^a$  look like the world-sheet heterotic fermions charged under the spacetime gauge group. The tension of the D string being inversely proportional to the string coupling constant, it is natural to conjecture that in the strong coupling regime it is the D string which governs the perturbative dynamics. This is similar to what happens in type I, where the D string is given by the  $SO(32)$  heterotic string [24]. The D string of the 0'B orientifold does not have boson-fermion degeneracy in neither the right nor the left sector so it cannot be described by a superconformal world-sheet theory. The quantization of this string theory is an open and interesting question. Let us just remark that the operators  $\theta^{\dagger\alpha}\theta^\beta$ ,  $\lambda^{\dagger a}\lambda^b\partial X^i$ ,  $\lambda^a\lambda^b\theta^\alpha$  are neutral under the  $U(1)$  two-dimensional gauge group and have the correct spacetime quantum numbers of respectively the Ramond-Ramond fields, the gauge bosons and the chiral fermions in the antisymmetric representation of  $U(32)$ .

## 4.5 The Wess-Zumino terms

For simplicity we concentrate, in this subsection, on the gauge anomalies. We would like to rederive in a unified fashion the gauge Wess-Zumino couplings we found in the previous subsection, revealing a remarkable structure of the 0'B D-branes. A concise way of writing all the Wess-Zumino interactions involving the gauge fields  $G$  and the normal curvature  $N$  is given by the formula [25]

$$\int A (\hat{A}(N))^{-1} \text{tr} e^{iG} , \quad (100)$$

where the roof genus  $\hat{A}(N)$  is defined in the Appendix (150) and where  $A$  is the formal sum of all the Ramond-Ramond forms,

$$A = \sum_{i=0}^8 T_{i-1} A_i . \quad (101)$$

It is understood that in (100) only the  $p + 1$  form term contributes for the Dp brane. Here, we shall give a direct proof of the gauge part in (100). Note first of all that the gauge anomaly in ten dimensions is given by the twelve-form part of  $Tr_A e^{iF}$ , where the trace is in the antisymmetric representation. In terms of the trace in the fundamental representation we have the simple formula

$$Tr_A e^{iF} = \frac{1}{2} \left[ (tr e^{iF})^2 - tr e^{2iF} \right] . \quad (102)$$

The term in  $tr F^6$  cancels in the above formula for  $U(32)$  since for  $U(N)$  the first term gives  $2N$  and the second  $-2^6$  times a common factor. The twelve form part of (102) can thus be written as the twelve form part of

$$\frac{1}{2} \left[ tr(e^{iF} - 1) \right]^2 , \quad (103)$$

which is of the factorised form

$$I_{12} = \frac{1}{2} \sum_{i=2}^8 X_i X_{12-i} . \quad (104)$$

In (104),  $X_i$  is the  $i$  form part of  $tr(e^{iF} - 1)$ ,

$$X_i = \left( tr(e^{iF} - 1) \right) |_{i} . \quad (105)$$

This reproduces exactly the gauge part of the anomaly polynomials (28). Next, note that all the D-branes have the following fermion content: positive chirality fermions in the antisymmetric of  $U(M)$ , negative chirality fermions in the symmetric of  $U(M)$ , both of them with multiplicity  $2^{(7-p)/2}$  and finally one multiplet of positive chirality fermions in the bifundamental of  $U(M) \times U(32)$ . Using the identity (102) with a similar one for the trace in the symmetric (which differs by the sign of the second term in (102)), we get for the gauge anomaly  $I_{p+3}$  of the Dp-brane the  $(p + 3)$  form part of

$$tr e^{iF} tr e^{iG} - 2^{\frac{p-7}{2}} tr e^{2iG} . \quad (106)$$

The term in  $tr G^{(p+3)/2}$  cancels in the above formula and so we have

$$I_{p+3} = tr(e^{iF} - 1) tr e^{iG} |_{p+3} , \quad (107)$$

which, using (105) can be written as

$$I_{p+3} = \sum_i X_i tr e^{iG} |_{p+3} . \quad (108)$$

This is exactly the anomaly compensated by (100). It is also possible, following the Appendix, to prove the form of the normal curvature part  $N$  of (100). Notice, however, that the  $R$  curvature part of the Wess-Zumino terms cannot be written in a simple compact form as the  $N$  part in (100).

## 5 Non-tachyonic compactifications on orbifolds

A generic compactification of  $O'B$  leads to the reappearance of tachyons. Consider for instance the circle compactification. The states  $|T, m, n\rangle \rightarrow -|T, m, -n\rangle$ , with KK momentum  $m$  and non-zero winding  $n$ , are invariant under  $\Omega'$  but are tachyonic for small radius and zero KK momenta  $m = 0$ . In the following we examine some examples which are free from tachyons.

### 5.1 A $9d$ example

The simplest non-tachyonic  $O'B$  compactification we have found is an orientifold of the freely-acting orbifold of  $OB$ , generated by the element  $(-1)^{F_L} \times I$ , where  $I$  is the shift operation  $IX_9 = X_9 + \pi r$  and  $X_9$  is the last (tenth) coordinate, of radius  $r$ . The torus amplitude reads

$$T = \sum_{m,n} \left\{ (|V_8|^2 + |S_8|^2) Z_{2m,n} + (|O_8|^2 + |C_8|^2) Z_{2m+1,n} - (V_8 \bar{S}_8 + S_8 \bar{V}_8) Z_{2m,n+\frac{1}{2}} - (O_8 \bar{C}_8 + C_8 \bar{O}_8) Z_{2m+1,n+\frac{1}{2}} \right\}, \quad (109)$$

where

$$Z_{m,n} = \frac{1}{|\eta(\tau)|^2} \sum_{m,n} q^{\frac{\alpha'}{4}(\frac{m}{r} + \frac{nr}{\alpha'})^2} \bar{q}^{\frac{\alpha'}{4}(\frac{m}{r} - \frac{nr}{\alpha'})^2}. \quad (110)$$

The amplitude (109) interpolates between type OB theory in the limit  $r \rightarrow \infty$  and type IIB theory in the  $r \rightarrow 0$  limit. In particular, for  $r > \sqrt{\alpha'}$  we recover the tachyonic state of type OB. It is however possible, starting from (109), to write down a Klein bottle amplitude which eliminates the closed tachyon for *all* radii<sup>8</sup>

$$K = \frac{1}{2} \left\{ (V_8 - S_8) \sum_m q^{\frac{(2m)^2}{R^2}} - (O_8 - C_8) \sum_m q^{\frac{(2m+1)^2}{R^2}} \right\} \frac{1}{\eta}. \quad (111)$$

Notice that in the  $R \rightarrow \infty$  limit and after an appropriate rescaling of  $T$  and  $K$ , the closed part of the model reproduces the 10d non-tachyonic  $O'B$  orientifold [6]. On the other hand, in the  $R \rightarrow 0$  limit we recover the closed part of Type I superstring. The model need for consistency the introduction of 32 D8 branes. This can be easily justified by writing the transverse Klein amplitude

$$\tilde{K} = 16R \left( \sum_n q^{(2n+1)^2 R^2} V_8 - \sum_n q^{(2n)^2 R^2} S_8 \right) \frac{1}{\eta}, \quad (112)$$

which manifests a RR tadpole asking for 32 D8 branes. In a spacetime interpretation, the model contains 16  $O8_+$  and 16  $O8_-$  planes. Since in the  $R \rightarrow 0$  limit the closed sector becomes supersymmetric, we can ask for a brane configuration

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<sup>8</sup>In the following,  $R$  denotes the dimensionless radius in string units.

in which the RR tadpole is *locally* cancelled in this limit, as in the M-theory compactifications with broken supersymmetry studied in [31]. The appropriate configuration has 16 D8 branes (of Chan-Paton charge  $N_1$ ) on top of the  $O8_+$  planes and 16 D8 branes (of Chan-Paton charge  $N_2$ ) on top of the  $O\bar{8}_-$  planes. The open sector amplitudes are

$$\begin{aligned} A &= \left\{ \frac{N_1^2 + N_2^2}{2} \sum_m q^{\frac{m^2}{R^2}} + N_1 N_2 \sum_m q^{\frac{(m+1/2)^2}{R^2}} \right\} (V_8 - S_8) \frac{1}{\eta} , \\ M &= \left\{ -\frac{N_1 - N_2}{2} \sum_m (-1)^m q^{\frac{m^2}{R^2}} V_8 + \frac{N_1 + N_2}{2} \sum_m q^{\frac{m^2}{R^2}} S_8 \right\} \frac{1}{\eta} . \end{aligned} \quad (113)$$

The global *and local* tadpole cancellation ask for  $N_1 = N_2 = 16$  and the resulting gauge group is  $SO(16) \times USp(16)$ . The massless spectrum contains also Majorana-Weyl fermions in  $(\mathbf{136}, \mathbf{1}) + (\mathbf{1}, \mathbf{136})$ . The resulting configuration contains the system 16  $O8_+ - 16 D8$  with (massless) supersymmetric spectrum at one of the orientifold fixed points, and the system 16  $O\bar{8}_- - 16 D8$  with non-supersymmetric spectrum at the other orientifold fixed point. The geometrical configuration is an interesting mixture between the Type I compactification called M-theory breaking in [31] and a 9d compactification of the Sugimoto model [27] with a particular Wilson line. Indeed, in the M-theory model, the geometric configuration in 9d contained two copies of the system 16  $O8_+ - 16 D8$ , one per orientifold fixed point, each carrying the gauge group  $SO(16)$  and a massless supersymmetric spectrum. On the other hand, by turning on a Wilson line, we can compactify to 9d the Sugimoto model and get a configuration with two copies of the system 16  $O\bar{8}_- - 16 D8$ , each giving rise to the gauge group  $USp(16)$ .

This compactification has (for any radius) no closed or open string tachyons and contains, as argued above, both the spectrum of M-theory breaking (or brane supersymmetry) [31] in one fixed point and the spectrum of brane supersymmetry breaking [28] in the other fixed point. It suggests also an interesting connection between the O'B orientifold introduced in [6] and the Sugimoto model.

It is natural to conjecture that, as the model has the property of satisfying *local* RR tadpole cancellation (notice the existence of a dilaton tadpole for all radii), by using arguments of the type [19, 31], this represents a non-supersymmetric compactification of M-theory down to 9d which breaks supersymmetry softly in the bulk and on one set of the D8 branes. As the model has no closed or open tachyons for all values of the radius, the configuration is stable at one-loop. This is to be contrasted with the case of M-theory breaking [31], where closed and open tachyons appear for critical value of the radius. It would be interesting to present the explicit M-theory compactification realizing this configuration.

## 5.2 O'B on $T^6/Z_2$

In constructing string amplitudes for D9-D(9-2n) branes, we compactify the non-tachyonic O'B orientifold [6] on the  $T^{2n}/Z_2$  orbifold ( $n = 1, 2, 3$ ), which reverses



the sign of the  $2n$  compact coordinates. The action in the RR sector is fixed by the action on the zero modes, which is  $Z_2 = \epsilon_n \prod_j (\Gamma \Gamma_j) \otimes \Gamma_j$ , where  $\Gamma$  is the 10d chirality matrix and  $\Gamma_j$  ( $j = 1 \cdots 2n$ ) are Dirac matrices. In addition,  $\epsilon_n$  are phases to be determined by consistency with the action on the open sector. The orbifold action on the  $SO(2n)$  characters is  $Z_2(O_{2n}, V_{2n}, S_{2n}, C_{2n}) = (O_{2n}, -V_{2n}, i^{n(2n-1)} S_{2n}, -i^{n(2n-1)} C_{2n})$ .

We consider here string propagation on the  $T^6/Z_2$  orientifold of Type O'B non-tachyonic model. Part of the D3 branes are moved together off the orbifold fixed points by some distance  $aR$ , creating images interchanged by the orbifold action. This allows a simple way of finding the D3 spectrum in the noncompact space by taking the limit of infinite compact volume and taking off the orbifold action.

We consider the orbifold acting on the 3 compact complex coordinates as  $Z_2 = (-1, -1, -1)$  and on the  $SO(6)$  characters (1) as  $Z_2(O_6, V_6, S_6, C_6) = (O_6, -V_6, -iS_6, iC_6)$ .

The corresponding orbifold torus amplitude, which is the starting point for the orientifold amplitudes below, is

$$\begin{aligned}
T = & \frac{1}{4} \left\{ \left( |O_2 O_6 + V_2 V_6|^2 + |O_2 V_6 + V_2 O_6|^2 + |S_2 S_6 + C_2 C_6|^2 + |S_2 C_6 + C_2 S_6|^2 \right) \right. \\
& \times \Gamma^{(6,6)} + \left( |O_2 O_6 - V_2 V_6|^2 + |O_2 V_6 - V_2 O_6|^2 - |S_2 S_6 - C_2 C_6|^2 - |S_2 C_6 - C_2 S_6|^2 \right) \left| \frac{2\eta}{\theta_2} \right|^6 \\
& + 64 \left( (O_2 S_6 + V_2 C_6) \overline{(O_2 C_6 + V_2 S_6)} + (O_2 C_6 + V_2 S_6) \overline{(O_2 S_6 + V_2 C_6)} \right. \\
& + (S_2 O_6 + C_2 V_6) \overline{(C_2 O_6 + S_2 V_6)} + (S_2 V_6 + C_2 O_6) \overline{(C_2 V_6 + S_2 O_6)} \left. \right) \left| \frac{\eta}{\theta_4} \right|^6 \\
& + 64 \left( (O_2 S_6 - V_2 C_6) \overline{(O_2 C_6 - V_2 S_6)} + (O_2 C_6 - V_2 S_6) \overline{(O_2 S_6 - V_2 C_6)} + \right. \\
& \left. (S_2 O_6 - C_2 V_6) \overline{(C_2 O_6 - S_2 V_6)} + (S_2 V_6 - C_2 O_6) \overline{(C_2 V_6 - S_2 O_6)} \right) \left| \frac{\eta}{\theta_3} \right|^6 \left. \right\}, \quad (114)
\end{aligned}$$

where  $\theta_i$  and  $\eta$  are the Jacobi and the Dedekind function, respectively and  $\Gamma^{(6,6)}$  is the  $6d$  compactification lattice given, in the simplest case of compact space described only by the radii  $r_i$ , by

$$\Gamma^{(6,6)} = \frac{1}{|\eta(\tau)|^{12}} \sum_{m_i, n_i} q^{\frac{\alpha'}{4} \sum_i (\frac{m_i}{r_i} + \frac{n_i r_i}{\alpha'})^2} \bar{q}^{\frac{\alpha'}{4} \sum_i (\frac{m_i}{r_i} - \frac{n_i r_i}{\alpha'})^2}. \quad (115)$$

There is another consistent, modular invariant torus amplitude, corresponding to an inverted orbifold action on the untwisted RR fields compared to (114), i.e.  $Z_2 |S_2 S_6 + C_2 C_6|^2 \rightarrow + |S_2 S_6 - C_2 C_6|^2$ . This orbifold action is however incompatible with the existence of an open sector. Indeed, as we will see below, the orbifold acts as  $\pm i$  on D3-D3 open fermions. Two such identical fermions can join and form a closed RR state of the type  $|S_2 S_6|^2$  or  $|C_2 C_6|^2$ , which must therefore be odd under the orbifold projection.

The Klein bottle amplitude of the orbifold in 4d, obtained from the torus amplitude (114) is

$$\begin{aligned}
K = & -\frac{1}{4} \left\{ \left[ \left( \sum_m q^{\frac{m^2}{R^2}} \right)^6 + \left( \sum_n q^{n^2 R^2} \right)^6 \right] (O_2 - V_2)(O_6 - V_6) \right. \\
& \left. + \left[ \left( \sum_m q^{\frac{m^2}{R^2}} \right)^6 - \left( \sum_n q^{n^2 R^2} \right)^6 \right] (S_2 - C_2)(S_6 - C_6) \right\} \frac{1}{\eta^6} . \quad (116)
\end{aligned}$$

In (116) and in what follows, we denote by  $m/R$  ( $nR$ ) Kaluza-Klein (winding) masses. For simplicity, we consider a compact space of common dimensionless radius  $R^2 = r^2/\alpha'$ , where  $r$  is the dimensionful radius and  $\alpha'$  is the string tension. Notice in (116) the absence of the twisted sector, equivalent in the transverse channel to the absence of O9-O3 orientifold amplitude. The amplitude (116) has RR tadpole divergences in the transverse channel which ask for consistency an open sector containing D9 and D3 branes.

We consider in the following the cylinder amplitude with D9 branes of Chan-Paton (complex) charge  $N$ , part of the D3 branes, of charge  $D$ , at one orbifold fixed point (the origin of the compact space, for simplicity) and the other D3 branes, of charge  $\delta$ , moved together in the bulk a distance  $aR$  away from the origin in pairs of orbifold images. The resulting amplitude reads

$$\begin{aligned}
A = & \frac{1}{2} \left\{ [N\bar{N} \left( \sum_m q^{\frac{m^2}{R^2}} \right)^6 + \sum_n ((D\bar{D} + 2\delta\bar{\delta})q^{n^2 R^2} + 2\delta\bar{\delta}(q^{(n+2a)^2 R^2} + q^{(n-2a)^2 R^2}) \right. \right. \\
& + (D\bar{\delta} + \bar{D}\delta)(q^{(n+a)^2 R^2} + q^{(n-a)^2 R^2})) \left( \sum_n q^{n^2 R^2} \right)^5] (O_2 V_6 + V_2 O_6) \frac{1}{\eta^6} \\
& - \frac{N^2 + \bar{N}^2}{2\eta^6} \left( \sum_m q^{\frac{m^2}{R^2}} \right)^6 (S_2 S_6 + C_2 C_6) - \frac{1}{2} \sum_n [(D^2 + \bar{D}^2 + 2(\delta^2 + \bar{\delta}^2))q^{n^2 R^2} + \\
& 2(D\delta + \bar{D}\bar{\delta})(q^{(n+a)^2 R^2} + q^{(n-a)^2 R^2}) + (\delta^2 + \bar{\delta}^2)(q^{(n+2a)^2 R^2} + q^{(n-2a)^2 R^2})] \\
& \times \left( \sum_n q^{n^2 R^2} \right)^5 (S_2 C_6 + C_2 S_6) \frac{1}{\eta^6} + [N(\bar{D} + 2\bar{\delta})(V_2 S_6 + O_2 C_6) + \bar{N}(D + 2\delta) \times \\
& (V_2 C_6 + O_2 S_6) - N(D + 2\delta)(C_2 V_6 + S_2 O_6) - \bar{N}(\bar{D} + 2\bar{\delta})(S_2 V_6 + C_2 O_6)] \left( \frac{\eta}{\theta_4} \right)^3 \\
& + (Tr\gamma_N Tr\gamma_{\bar{N}} + Tr\gamma_D Tr\gamma_{\bar{D}})(V_2 O_6 - O_2 V_6) \left( \frac{2\eta}{\theta_2} \right)^3 + \frac{i}{2} \times \\
& \left[ ((Tr\gamma_N)^2 + (Tr\gamma_{\bar{N}})^2)(S_2 S_6 - C_2 C_6) - ((Tr\gamma_D)^2 + (Tr\gamma_{\bar{D}})^2)(S_2 C_6 - C_2 S_6) \right] \\
& \times \left( \frac{2\eta}{\theta_2} \right)^3 + [-iTr\gamma_N Tr\gamma_{\bar{D}}(V_2 S_6 - O_2 C_6) + iTr\gamma_{\bar{N}} Tr\gamma_D(V_2 C_6 - O_2 S_6) + \\
& Tr\gamma_N Tr\gamma_D(C_2 V_6 - S_2 O_6) + Tr\gamma_{\bar{N}} Tr\gamma_{\bar{D}}(S_2 V_6 - C_2 O_6)] \left( \frac{\eta}{\theta_3} \right)^3 \left. \right\} , \quad (117)
\end{aligned}$$

where  $\gamma_N$  ( $\gamma_D$ ) denotes the  $Z_2$  orbifold action on D9 brane (D3 at the orbifold fixed point) Chan-Paton matrices. Notice the peculiar structure of cylinder amplitudes,

in particular the internal chirality of fermions in the D9-D9 and D3-D3 sector are opposite, whereas in the D9-D3 sector complex conjugation in the Chan-Paton sector exchanges conjugate characters  $S_{2,6} \leftrightarrow C_{2,6}$ .

The Möbius amplitude reads

$$\begin{aligned}
M = & \frac{1}{4} \left\{ (N + \bar{N}) \left( \sum_m q^{\frac{m^2}{R^2}} \right)^6 (S_2 S_6 + C_2 C_6) \frac{1}{\eta^6} + \sum_n [(D + \bar{D}) q^{n^2 R^2} + \right. \\
& (\delta + \bar{\delta}) (q^{(n+2a)^2 R^2} + q^{(n-2a)^2 R^2})] \left( \sum_n q^{n^2 R^2} \right)^5 (S_2 C_6 + C_2 S_6) \frac{1}{\eta^6} \\
& \left. + [(N - \bar{N}) (S_2 S_6 - C_2 C_6) + (D - \bar{D} + 2\delta - 2\bar{\delta}) (S_2 C_6 - C_2 S_6)] \left( \frac{2\eta}{\theta_2} \right)^3 \right\} \quad (118)
\end{aligned}$$

We checked that the amplitudes (116)-(118) satisfy the rules for orientifold construction: RR tadpole cancellation, factorization of amplitudes in the transverse channel and spacetime particle interpretation. In the compact version of the model, RR tadpole cancellation asks for the conditions

$$\begin{aligned}
& \text{untwisted : } N + \bar{N} = 64, \quad D + \bar{D} + 2(\delta + \bar{\delta}) = 64, \\
& \text{twisted : } Tr \gamma_N = 0, \quad Tr \gamma_D = 0.
\end{aligned} \quad (119)$$

This, together with the appropriate particle interpretation, fixes the parametrization

$$N = n_1 + n_2, \quad D = d_1 + d_2, \quad (120)$$

where  $n_{1,2}$  and  $d_{1,2}$  are complex charges. The RR tadpole conditions (119) give  $n_1 = n_2 = 16$  and  $d_1 = d_2 = d$  and therefore the orbifold action on the D9 and D3 (at the orbifold fixed point) Chan-Paton charges is

$$\gamma_N = e^{\frac{i\pi}{4}} (I_{16}, -I_{16}), \quad \gamma_D = e^{\frac{-i\pi}{4}} (I_d, -I_d), \quad (121)$$

where  $I_n$  is the  $n \times n$  identity matrix. The  $\exp(i\pi/4)$  phase in the orbifold action is needed in order for the complete orbifold action on the open string fermions to be a  $Z_2$  action<sup>9</sup>. Indeed, the orbifold action on the 99 fermion zero modes is  $\pm i$  and the phase in (121) gives an additional action of  $i$  on the Chan-Paton factors.

The resulting gauge group is  $[U(16) \times U(16)]_9 \times [[U(d) \times U(d)]_3 \times U(\delta)_3]$ , where  $d + \delta = 16$ . Notice the usual rank reduction of the D3 gauge group due to the orbifold, which exchanges D3 branes with their orbifold mirrors. The massless spectrum in four dimensions consists of

$$\begin{aligned}
& 6 \text{ scalars : } (\mathbf{16}, \overline{\mathbf{16}}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\overline{\mathbf{16}}, \mathbf{16}; \mathbf{1}, \mathbf{1}, \mathbf{1}) \\
& + (\mathbf{1}, \mathbf{1}; \mathbf{d}, \bar{\mathbf{d}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \bar{\mathbf{d}}, \mathbf{d}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \delta\bar{\delta}),
\end{aligned}$$

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<sup>9</sup>This was already noticed in [26], where the spectrum of  $T^6/Z_2$  OB orientifold was worked out in the compact space and with all D3 branes at the orbifold fixed point.

$$\begin{aligned}
& 4 \text{ Weyl fermions : } (\mathbf{120}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{120}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \frac{\mathbf{d}(\mathbf{d}-1)}{2}, \mathbf{1}, \mathbf{1}) \\
& + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \frac{\mathbf{d}(\mathbf{d}-1)}{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \frac{\delta(\delta-1)}{2}) + (\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \frac{\overline{\delta(\delta+1)}}{2}) \\
& + (\overline{\mathbf{16}}, \overline{\mathbf{16}}; \mathbf{1}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}; \bar{\mathbf{d}}, \bar{\mathbf{d}}, \mathbf{1}) , \\
& \text{Weyl fermions : } (\mathbf{16}, \mathbf{1}; \mathbf{d}, \mathbf{1}, \mathbf{1}) + (\mathbf{1}, \mathbf{16}; \mathbf{1}, \mathbf{d}, \mathbf{1}) \\
& + (\mathbf{16}, \mathbf{1}; \mathbf{1}, \mathbf{1}, \delta) + (\mathbf{1}, \mathbf{16}; \mathbf{1}, \mathbf{1}, \delta) . \tag{122}
\end{aligned}$$

The effective field theory interactions between D-branes/O-planes and the massless closed fields in this model it is easily found by looking at the transverse string amplitudes corresponding to the tree-level propagation of massless closed states. By using (116)-(118) and denoting by  $\tilde{K}_0$ ,  $\tilde{A}_0$ ,  $\tilde{M}_0$  the corresponding amplitudes, we find, omitting unphysical couplings (for ex. proportional to  $N - \bar{N}$ ), the result

$$\begin{aligned}
\tilde{K}_0 + \tilde{A}_0 + \tilde{M}_0 = & \frac{2^{-5}}{8} \left\{ [(N + \bar{N})\sqrt{v} - \frac{D + \bar{D} + 2\delta + 2\bar{\delta}}{\sqrt{v}}]^2 (O_2 V_6)_0 + \right. \\
& \left. [(N + \bar{N})\sqrt{v} + \frac{D + \bar{D} + 2\delta + 2\bar{\delta}}{\sqrt{v}}]^2 (V_2 O_6)_0 \right. \\
& \left. - (N + \bar{N} - 64)^2 v (S_2 S_6 + C_2 C_6)_0 - \frac{(D + \bar{D} + 2\delta + 2\bar{\delta} - 64)^2}{v} (S_2 C_6 + C_2 S_6)_0 \right\} \tag{123}
\end{aligned}$$

where the subscript 0 on the various characters retains only their massless part. The corresponding effective interactions read therefore

$$\begin{aligned}
S_{\text{int}} = & -\frac{1}{2} T_9 \int d^{10}x \{ \sqrt{-g} (N + \bar{N}) e^{-\Phi} + (N + \bar{N} - 64) A_{10} \} \\
& - \frac{1}{2} T_3 \int d^4x \{ \sqrt{-g} [D + \bar{D} + 2(\delta + \bar{\delta})] e^{-\Phi} + [D + \bar{D} + 2(\delta + \bar{\delta}) - 64] A_4 \} , \tag{124}
\end{aligned}$$

where  $A_{10}$  ( $A_4$ ) is the ten-form (four-form) coupling to D9 (D3) branes. The terms containing Chan-Paton factors  $N, D$  (numerical factors with 64) describe the interactions of D9 and D3 branes (O9 and O3 planes) with closed fields. Notice, in analogy with the 10d counterpart (27), the unavoidable presence of the dilaton tadpoles, present generically in most orientifold models with broken supersymmetry [27], [28]. They ask for a background redefinition [29], explicitly performed recently [30] for the 10d model introduced in [27].

In the case of D3 branes in noncompact space, RR tadpoles give no constraints on the rank of the D3 gauge group. The total gauge group becomes  $U(32)_9 \times U(M)_3$ . The matter spectrum of the noncompact model in 4d includes the massless particles

$$\begin{aligned}
& 6 \text{ scalars : } (\mathbf{1024}; \mathbf{1}) + (\mathbf{1}; \mathbf{M}\bar{\mathbf{M}}), \\
& 4 \text{ Weyl fermions : } (\mathbf{496}; \mathbf{1}) + (\overline{\mathbf{496}}; \mathbf{1}) + (\mathbf{1}; \frac{\mathbf{M}(\mathbf{M}-1)}{2}) + (\mathbf{1}; \frac{\overline{\mathbf{M}(\mathbf{M}+1)}}{2}) , \\
& \text{Weyl fermions : } (\mathbf{32}; \mathbf{M}) . \tag{125}
\end{aligned}$$

Notice that this spectrum agrees with the one obtained in the Section 4.1 by using a realization of the orientifold projection on the Chan-Paton factors by matrices. It is also interesting to notice the bose-fermi degeneracy in the 33 massless spectrum.

### 5.3 0'B on $T^4/Z_2$

We consider here the  $T^4/Z_2$  OB compact orbifold acting on the two complex compact coordinates as  $Z_2(z_1, z_2) = (-z_1, -z_2)$  and on the  $SO(4)$  characters as  $Z_2(O_4, V_4, S_4, C_4) = (O_4, -V_4, -S_4, C_4)$ . The open spectrum contains D9 and D5 branes. The case with D5 branes at the orbifold fixed points was considered in [7]. We consider here the straightforward extension of taking *all* D5 branes off the fixed points which encodes the spectrum of D5 branes in the noncompact space.

The corresponding orbifold torus amplitude of the model reads

$$\begin{aligned}
T = & \frac{1}{4} \left\{ \left( |O_4 O_4 + V_4 V_4|^2 + |O_4 V_4 + V_4 O_4|^2 + |S_4 S_4 + C_4 C_4|^2 + \right. \right. \\
& \left. |S_4 C_4 + C_4 S_4|^2 \right) \Gamma^{(4,4)} + \left( |O_4 O_4 - V_4 V_4|^2 + |O_4 V_4 - V_4 O_4|^2 + \right. \\
& \left. |S_4 S_4 - C_4 C_4|^2 + |S_4 C_4 - C_4 S_4|^2 \right) \left| \frac{2\eta}{\theta_2} \right|^4 + 16 \left( |O_4 S_4 + V_4 C_4|^2 \right. \\
& + |O_4 C_4 + V_4 S_4|^2 + |S_4 O_4 + C_4 V_4|^2 + |S_4 V_4 + C_4 O_4|^2 \left. \right) \left| \frac{\eta}{\theta_4} \right|^4 + \\
& 16 \left( |O_4 S_4 - V_4 C_4|^2 + |O_4 C_4 - V_4 S_4|^2 + \right. \\
& \left. |S_4 O_4 - C_4 V_4|^2 + |S_4 V_4 - C_4 O_4|^2 \right) \left| \frac{\eta}{\theta_3} \right|^4 \left. \right\}, \tag{126}
\end{aligned}$$

where  $\Gamma^{(4,4)}$  is the  $4d$  compactification lattice.

The Klein amplitude in 6d is

$$\begin{aligned}
K = & -\frac{1}{4} \left\{ \left[ \left( \sum_m q^{\frac{m^2}{R^2}} \right)^4 + \left( \sum_n q^{n^2 R^2} \right)^4 \right] [(O_4 - V_4)(O_4 - V_4) + (S_4 - C_4)(S_4 - C_4)] \frac{1}{\eta^4} \right. \\
& \left. + 32[(O_4 - V_4)(S_4 - C_4) + (S_4 - C_4)(O_4 - V_4)] \left( \frac{\eta}{\theta_4} \right)^2 \right\}. \tag{127}
\end{aligned}$$

The cylinder amplitudes with all D5 branes moved together off the fixed points is

$$\begin{aligned}
A = & \frac{1}{2} \left\{ [N \bar{N} \left( \sum_m q^{\frac{m^2}{R^2}} \right)^4 + 2D \bar{D} \sum_n (q^{n^2 R^2} + q^{(n+2a)^2 R^2} + q^{(n-2a)^2 R^2}) \left( \sum_n q^{n^2 R^2} \right)^3] \right. \\
& \times (O_4 V_4 + V_4 O_4) \frac{1}{\eta^4} - \frac{1}{2} [(N^2 + \bar{N}^2) \left( \sum_m q^{\frac{m^2}{R^2}} \right)^4 + (D^2 + \bar{D}^2) \sum_n (2q^{n^2 R^2} + q^{(n+2a)^2 R^2} \\
& \left. + q^{(n-2a)^2 R^2}) \left( \sum_n q^{n^2 R^2} \right)^3] \left. \right\}
\end{aligned}$$

$$\begin{aligned}
& + q^{(n-2a)^2 R^2} \left( \sum_n q^{n^2 R^2} \right)^3 \left( S_4 S_4 + C_4 C_4 \right) \frac{1}{\eta^4} + 2[(N\bar{D} + D\bar{N})(V_4 S_4 + O_4 C_4) \\
& - (ND + \bar{N}\bar{D})(C_4 V_4 + S_4 O_4)] \left( \frac{\eta}{\theta_4} \right)^2 + Tr \gamma_N Tr \gamma_{\bar{N}} (V_4 O_4 - O_4 V_4) \left( \frac{2\eta}{\theta_2} \right)^2 \\
& + \frac{1}{2} [(Tr \gamma_N)^2 + (Tr \gamma_{\bar{N}})^2] (S_4 S_4 - C_4 C_4) \left( \frac{2\eta}{\theta_2} \right)^2 \Big\} , \tag{128}
\end{aligned}$$

where here  $N$  ( $D$ ) denotes D9 (D5) brane Chan-Paton charge. The associated Möbius amplitude is

$$\begin{aligned}
M = & \frac{1}{4} \Big\{ [(N + \bar{N}) \left( \sum_m q^{\frac{m^2}{R^2}} \right)^4 + (D + \bar{D}) \sum_n (q^{(n+2a)^2 R^2} + q^{(n-2a)^2 R^2}) \left( \sum_n q^{n^2 R^2} \right)^3] \\
& \times (S_4 S_4 + C_4 C_4) \frac{1}{\eta^4} + (N + \bar{N} + 2D + 2\bar{D}) (S_4 S_4 - C_4 C_4) \left( \frac{2\eta}{\theta_2} \right)^2 \Big\} . \tag{129}
\end{aligned}$$

The RR tadpoles of the model ask for  $N + \bar{N} = 64$ ,  $D + \bar{D} = 32$  and the twisted tadpoles for  $\gamma_N = (iI_{16}, -iI_{16})$ . The spectrum of the model in the compact space with D5 branes off the fixed points is a straightforward generalization of the one worked out in [7], in particular the gauge group is  $[U(16) \times U(16)]_9 \times U(16)_5$ .

In the noncompact version, the gauge group becomes  $U(32)_9 \times U(M)_5$ . The massless charged matter content in six dimensions is

$$\begin{aligned}
& 4 \text{ scalars} : (\mathbf{1024}; \mathbf{1}) + (\mathbf{1}; \mathbf{M}\bar{\mathbf{M}}), \\
& \text{Weyl}_+ \text{ fermions} : (\mathbf{496}; \mathbf{1}) + (\overline{\mathbf{496}}; \mathbf{1}) + 2 \times (\mathbf{1}; \frac{\mathbf{M}(\mathbf{M}-1)}{2}) , \\
& \text{Weyl}_- \text{ fermions} : (\mathbf{496}; \mathbf{1}) + (\overline{\mathbf{496}}; \mathbf{1}) + 2 \times (\mathbf{1}; \frac{\mathbf{M}(\mathbf{M}+1)}{2}) , \\
& 2 \text{ complex scalars} : (\mathbf{32}; \bar{\mathbf{M}}) , \\
& \text{Weyl}_+ \text{ fermions} : (\mathbf{32}; \mathbf{M}) . \tag{130}
\end{aligned}$$

This spectrum agrees with the one previously found in Section 4.3. Notice again the fermi-bose degeneracy in the 55 massless spectrum.

## 5.4 O'B on $T^2/Z_2$

The D9-D7 brane spectrum is found by starting with the compact  $T^2/Z_2$  orbifold containing D9 and D7 branes and then extending the results to the noncompact space. The analysis is very similar to the  $T^6/Z_2$  orbifold considered in the D9-D3 case and will not repeated here. The gauge group in the compact space case, with all D7 branes moved together off the orbifold fixed points is  $[U(16) \times U(16)]_9 \times U(16)_7$ .

In the case of D7 branes in noncompact space, RR tadpoles give no constraints on the rank of the D7 gauge group. The total gauge group becomes  $U(32)_9 \times$

$U(M)_7$ . The massless spectrum of the noncompact model in 8d includes

$$\begin{aligned}
& 2 \text{ scalars : } (\mathbf{1024}; \mathbf{1}) + (\mathbf{1}; \mathbf{M}\bar{\mathbf{M}}), \\
& \text{Weyl fermions : } (\mathbf{496}; \mathbf{1}) + (\overline{\mathbf{496}}; \mathbf{1}) + (\mathbf{1}; \frac{\mathbf{M}(\mathbf{M}-1)}{2}) + (\mathbf{1}; \frac{\overline{\mathbf{M}(\mathbf{M}+1)}}{2}) , \\
& \text{Weyl fermions : } (\mathbf{32}; \mathbf{M}) .
\end{aligned} \tag{131}$$

In this case, however, there are tachyonic scalars in the 97 sector, in the representations  $(\mathbf{32}; \bar{\mathbf{M}}) + (\overline{\mathbf{32}}; \mathbf{M})$ . Their presence suggests a tachyon condensation that results, for  $M < 32$ , in the unbroken gauge group  $U(32 - M)_9$ , while for  $M > 32$  the unbroken gauge group is  $U(M - 32)_7$ . This spectrum agrees with the one found in Section 4.2 by using a realization of orientifold projection on Chan-Paton factors by matrices. As in the previous cases, there is a bose-fermi degeneracy in the 77 massless spectrum.

## 6 Classical background of D9-D3 brane system and AdS/CFT correspondence

The effective action of the system in noncompact space and  $D3$  gauge group  $U(M)$  is easily extracted from (124), by ignoring the O3 plane contribution:

$$S_{\text{int}} = -32T_9 \int d^{10}x \sqrt{-g} e^{-\Phi} - MT_3 \int d^4x \{ \sqrt{-g} e^{-\Phi} + A_4 \} . \tag{132}$$

In order to derive the classical background, we follow [4] which parametrize the metric (in the Einstein frame) as

$$ds^2 = e^{\frac{1}{2}\xi(\rho) - 5\eta(\rho)} d\rho^2 + e^{-\frac{1}{2}\xi(\rho)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{\frac{1}{2}\xi(\rho) - \eta(\rho)} d\Omega_5^2 , \Phi = \Phi(\rho) . \tag{133}$$

As explained in [4], the field equations can be found from the analog one-dimensional Lagrangian system

$$S = \int d\rho \left[ \frac{1}{2} \left( \frac{\partial \Phi}{\partial \rho} \right)^2 + \frac{1}{2} \left( \frac{\partial \xi}{\partial \rho} \right)^2 - 5 \left( \frac{\partial \eta}{\partial \rho} \right)^2 - V \right] , \tag{134}$$

supplemented by the constraint

$$\frac{1}{2} \left( \frac{\partial \Phi}{\partial \rho} \right)^2 + \frac{1}{2} \left( \frac{\partial \xi}{\partial \rho} \right)^2 - 5 \left( \frac{\partial \eta}{\partial \rho} \right)^2 + V = 0 . \tag{135}$$

In our case (132), the potential  $V$  is

$$V = -\alpha e^{\frac{3\Phi}{2} + \frac{1}{2}\xi - 5\eta} + 20 e^{-4\eta} - M^2 e^{-2\xi} , \tag{136}$$

where  $M$  is the number of D3 branes in noncompact space and the first term in  $V$  comes from the 10d dilaton tadpole in (132), with  $\alpha = 32T_9\alpha'^5$ . The field equations derived from (134) read

$$\begin{aligned}\Phi'' &= \frac{3\alpha}{2}e^{\frac{3\Phi}{2}+\frac{1}{2}\xi-5\eta}, \quad \xi'' = \frac{\alpha}{2}e^{\frac{3\Phi}{2}+\frac{1}{2}\xi-5\eta} - 2M^2e^{-2\xi}, \\ \eta'' &= \frac{\alpha}{2}e^{\frac{3\Phi}{2}+\frac{1}{2}\xi-5\eta} - 8e^{-4\eta},\end{aligned}\tag{137}$$

where  $\Phi'' \equiv \partial^2\Phi/\partial\rho^2$ , etc. There is no known analytical solution for the coupled system of eqs. (137), however approximate solutions can be found in the IR ( $\rho \rightarrow \infty$ ) and in the UV ( $\rho \rightarrow 0$ ). Notice first of all that, from (137) the dilaton increases at larger distances from the brane. This suggests that the gauge theory is strong in the UV, and we will indeed check this prediction later on.

For large number of D3 branes, the tadpole contribution can be neglected and an approximate solution is [4]

$$\Phi^{(0)} = \Phi_0, \quad e^{\xi^{(0)}} = e^{\Phi_0} + \sqrt{2}M\rho, \quad e^{\eta^{(0)}} = 2\sqrt{\rho}.\tag{138}$$

In the near horizon limit, this reduces to the  $AdS_5 \times S^5$  background. The AdS/CFT correspondence identifies the energy scale  $u$  of the gauge theory to  $u^2 = \exp(-\xi/2)$ .

In the next approximation, we can compute the linear fluctuation around this solution due to the tadpole

$$\Phi = \Phi^{(0)} + \Phi^{(1)}, \quad \xi = \xi^{(0)} + \xi^{(1)}, \quad \eta = \eta^{(0)} + \eta^{(1)}.\tag{139}$$

In the IR limit ( $u \ll 1$ )  $\rho \rightarrow \infty$ , a straightforward computation gives

$$\begin{aligned}\Phi^{(1)} &= -\frac{3\alpha}{64}(\sqrt{2}M)e^{\frac{3\Phi_0}{2}}\ln\rho, \\ \xi^{(1)}(\rho) &= -\frac{\alpha}{128}e^{\frac{3\Phi_0}{2}}(\sqrt{2}M)^{\frac{1}{2}} + \frac{C}{\rho}, \\ \eta^{(1)}(\rho) &= -\frac{\alpha}{128}e^{\frac{3\Phi_0}{2}}(\sqrt{2}M)^{\frac{1}{2}} + \frac{C'}{\rho},\end{aligned}\tag{140}$$

where  $\Phi_0, C$  and  $C'$  are integration constants. We did choose some other integration constants in (140) in order to keep the linear fluctuations small in the IR.

The physical gauge coupling is  $1/g_{YM}^2 = M\exp(\Phi)$ , which is approximately given, in the IR region, by neglecting for simplicity the linear fluctuation in  $\xi$ , by

$$\frac{1}{g_{YM}^2} = e^{-\Phi_0} - \frac{3\alpha}{16M}(\sqrt{2}Me^{\Phi_0})^{\frac{1}{2}}\ln u.\tag{141}$$

The logarithmic evolution of the gauge couplings, coming from the dilaton tadpole and noticed previously in [9] is remarkable, since is qualitatively similar to the



renormalization group running of the D3 brane gauge theory. Notice, however, that since  $\alpha > 0$ , the gauge theory is predicted to be free in the IR and not in the UV, as previously thought. This prediction can be readily checked by computing the gauge theory one-loop beta function for the D3 gauge theory using the spectrum displayed in (125) and in Section 4.1. Notice first of all from (81) that the  $U(1)$  gauge boson in  $U(M)$  mixes with the two-form  $A_2$  and becomes massive, leaving an  $SU(M)$  unbroken gauge group. Using standard formulae, we then find the one-loop beta function

$$b_{SU(M)} = \frac{11}{3}T_G - \frac{2}{3}\sum_f T_f - \frac{1}{3}\sum_s T_s = \frac{11}{3}M - \frac{8}{3}\left(\frac{M-2}{2} + \frac{M+2}{2}\right) - \frac{2}{3} \times \frac{32}{2} - \frac{1}{6} \times 6M = -\frac{32}{3}, \quad (142)$$

where  $T_G$  denotes the Dynkin index for the adjoint representation and  $T_f$  ( $T_s$ ) is the Dynkin index for Weyl fermion (complex scalar) representations. The result (142) indeed confirms the sign and the  $1/M$  dependence appearing on the gauge theory side, found by taking the large  $M$  limit with fixed  $M \exp(\Phi)$ . Moreover, the connection between the gauge theory one-loop beta function and the dilaton tadpole is transparent, since because on the gauge theory side the D3-D3 spectrum is conformal at one-loop and only the D9-D3 spectrum, containing Weyl fermions in the representation  $(\mathbf{32}, \mathbf{M})$ , contributes. The numerical factor 32 is here the rank of the D9 brane gauge group  $U(32)$ , which determines also the dilaton tadpole (124). The precise numerical coefficient, however, in (141) is different from the one appearing in (142). This is due to the fact, noticed in [4], that corrections to (141) are of the same order as the leading result.

In the UV region  $u \gg 1$ , the gauge coupling becomes strong. Field eqs. (137) can be analytically solved if we retain only the tadpole contribution. However, the solution depends on a large number of integration constants which should be fixed by matching the IR solution (138), (140) with the UV one. This seems hard to realize, but it would be interesting to see the fate of the Landau pole on the gravity side.

## 7 Conclusion

In this paper we have determined the spectrum of D-branes in the non-tachyonic 0'B orientifold using two methods: the first is based on solving the consistency conditions which are essentially  $\Omega^2 = 1$  on the tensor product of the Chan-Paton and oscillator degrees of freedom and correct transverse channel interpretation. The second consists in considering compactifications on  $T^{2n}/Z_2$  where in order to cancel the tadpoles one has to introduce D-branes. By moving the branes from the fixed points and taking the infinite volume limit one gets the corresponding branes in non-compact space.

We have found that all the D branes are characterised by world-volume  $U(M)$  gauge groups with a chiral fermion content. There are massless Weyl fermions in the symmetric, antisymmetric and fundamental representations of  $U(M)$ , the chiralities for the antisymmetric and fundamental representations being the same. We verified that the world-volume anomalies associated to the gauge and gravitational anomalies due to the chiral fermions are precisely those requested by the form of the ten-dimensional anomaly and by the Green-Schwarz mechanism. This also allowed us to predict new Wess-Zumino couplings of the D-branes.

We also found a 9d compactification which in the closed sector interpolates between IIB and OB. By choosing a Klein projection similar to the 10d O'B model [6], we eliminate the closed string tachyon for *any* radius. The open sector is tachyon free for all radii, too, and has a spectrum combining in an interesting way supersymmetry breaking in the bulk (M-theory breaking [31]) and supersymmetry breaking on the branes (brane supersymmetry breaking [28]).

Another relation between the world-volume and spacetime theories is the Maldacena conjecture that we considered for the D-3 branes. The gauge theory of the latter has a positive beta function which is independent of  $N$  at one loop. We related the running of the gauge coupling constant to the classical variation of the dilaton due to the disk tadpole, with a good qualitative agreement.

Due to the Ramond-Ramond charges carried and the non-tachyonic nature of their spectrum, the D branes discussed in this paper are expected to be stable, even if they are not BPS. However, the existence of the NS-NS tadpoles and also the nonvanishing of the one-loop vacuum energy means that the perturbative vacuum is not the true ground state. In particular, this means that while for BPS branes, far away (with the exception of D7 and D9 branes) the background is asymptotic to the Minkowski space, this is not the case for the type of non-BPS branes discussed in our paper. One possibility (in the case of non-compact space) is to take a large number of branes, in which case the tadpoles and one-loop vacuum energy are subleading in most of the transverse space. Another one is to find the explicit background redefinition, in the spirit of the Fishler-Susskind mechanism [29], as in the 10d example worked recently in [30]. This second possibility, discussed briefly in Section 6, together with the quantization around the new background, clearly deserves further attention and work.

## Acknowledgments

We are grateful to C. Angelantonj and A. Sagnotti for useful discussions and comments. E.D. would like to thank the Aspen Center for Physics for hospitality and J.M. is grateful to the Center for Advanced Mathematical Sciences in Beirut, where part of this work was carried out.

## A The Wess-Zumino terms of the D7 and D5 branes

The anomaly polynomial of the  $D7$ -brane is given by

$$I_{10} = \hat{A}(\Sigma)[e^{\frac{N}{2}} Tr_A(e^{iG}) - e^{-\frac{N}{2}} Tr_S(e^{iG}) + tr(e^{iF})tr(e^{iG})] . \quad (143)$$

In this Appendix we explain some steps that lead to the factorised form (84) and determine explicitly the terms dependent on the normal curvature  $N$  in the  $Y_i$ . First, we note that  $\hat{A}(\Sigma)$  can be written as

$$\hat{A}(\Sigma) = \hat{A}(R) \left( \hat{A}(N) \right)^{-1} , \quad (144)$$

where  $R$  is the pull-back of spacetime curvature on the brane. Since the normal bundle is one-dimensional, the roof genus  $\hat{A}(N)$  is given by

$$\hat{A} = \frac{N/2}{\sinh(N/2)} . \quad (145)$$

Using this relation, as well as

$$Tr_A^S e^{iG} = \frac{1}{2} \left[ (tr e^{iG})^2 \pm tr(e^{2iG}) \right] , \quad (146)$$

the anomaly (143) can be put in the form

$$I_{10} = \hat{A}(R) \left[ tr e^{iG} \frac{\sinh(N/2)}{N/2} \left( tr e^{iF} + \frac{N}{2} \frac{\sinh(N/2)}{N/2} \right) - tr e^{2iG} \frac{\sinh(N)}{N} \right] . \quad (147)$$

If we define  $J = tr e^{iG} (2/N) \sinh(N/2) = \sum J_{2i}$ , where  $J_{2i}$  is the  $2i$  form part, we have

$$tr e^{2iG} \frac{\sinh(N)}{N} = \sum 2^i J_{2i} . \quad (148)$$

Inserting this into (147), we get

$$I_{10} = \hat{A}(R) \left[ J \left( tr e^{iF} + \frac{N}{2} J \right) - \sum 2^i J_{2i} \right] . \quad (149)$$

Now we use the expansion of the roof genus

$$\hat{A} = 1 - \frac{p_1}{24} + \frac{7p_1^2 - 4p_2}{5760} + \dots , \quad (150)$$

where we defined the first two Pontryagin classes

$$p_1(R) = -\frac{1}{2} tr R^2 , \quad p_2(R) = \frac{1}{8} (tr R^2)^2 - \frac{1}{4} tr R^4 . \quad (151)$$

By using also the expression of the polynomials  $X_i$  displayed in (29)

$$\begin{aligned} X_8 &= \frac{1}{4!} \text{tr} F^4 - \frac{1}{48} p_2 + \frac{1}{64} p_1^2 + \frac{1}{96} \text{tr} F^2 p_1 , \\ X_{10} &= \frac{i}{5!} \text{tr} F^5 - \frac{1}{2880} i \text{tr} F p_2 - \frac{1}{3840} i \text{tr} F p_1^2 , \end{aligned} \quad (152)$$

we can put the anomaly in the form

$$\begin{aligned} I_{10} &= M X_{10} + J_2 X_8 + (J_4 - M \frac{p_1}{24}) X_6 + (J_6 - J_2 \frac{p_1}{48}) X_4 \\ &+ (J_8 - \frac{M}{2880} p_2 - \frac{M}{3840} p_1^2) X_2 \\ &+ N (M J_8 + J_2 J_6 + \frac{1}{2} J_4^2 - \frac{M}{24} J_4 p_1 - \frac{1}{48} J_2^2 p_1 + M^2 \frac{7p_1^2 - 4p_2}{5760}) . \end{aligned} \quad (153)$$

From the eq. (153), we deduce

$$\begin{aligned} Y_2 &= J_2 = i \text{tr} G, \quad Y_4 = J_4 - M \frac{p_1}{24} = -\frac{1}{2} \text{tr} G^2 + \frac{M}{24} N^2 + \frac{M}{48} \text{tr} R^2 , \\ Y_6 &= J_6 - J_2 \frac{p_1}{48} = -\frac{i}{6} \text{tr} G^3 + \frac{i}{24} \text{tr} G N^2 + \frac{i}{96} \text{tr} G \text{tr} R^2 , \\ Y_8 &= J_8 - \frac{M}{2880} p_2 - \frac{M}{48 \times 80} p_1^2 = \\ &\frac{1}{24} \text{tr} G^4 - \frac{1}{48} \text{tr} G^2 N^2 + \frac{M}{1920} N^4 + \frac{M}{11520} \text{tr} R^4 - \frac{M}{9216} (\text{tr} R^2)^2 . \end{aligned} \quad (154)$$

The term which multiplies  $N$  in the last line in (153) can be put in the form

$$M Y_8 + Y_2 Y_6 + \frac{1}{2} Y_4^2 , \quad (155)$$

which completes the derivation of (84).

The anomaly of the 5-brane reads

$$I_6 = \hat{A}(\Sigma) [ch_+(N) \text{Tr}_A(e^{iG}) - ch_-(N) \text{Tr}_S(e^{iG}) + \text{tr}(e^{iF}) \text{tr}(e^{iG})] . \quad (156)$$

Let  $\lambda_1$  and  $\lambda_2$  be the Chern roots of the curvature in the fundamental representation of  $SO(4)$ , then

$$ch(S_\pm) = e^{(\frac{\lambda_1 \pm \lambda_2}{2})} + e^{-(\frac{\lambda_1 \pm \lambda_2}{2})} . \quad (157)$$

The first Pontryagin class is given by  $p_1 = \lambda_1^2 + \lambda_2^2$ , the Euler class is given by  $\chi = \lambda_1 \lambda_2$  and the second Pontryagin class by  $p_2 = \lambda_1^2 \lambda_2^2$ . Therefore we get the Chern characters of the spin bundle

$$ch(S_\pm) = 2 + \frac{(p_1 \pm 2\chi)}{4} + \frac{p_1^2 + 4p_2 \pm 4p_1 \chi}{192} . \quad (158)$$

The roof genus of the normal bundle is now given by

$$\hat{A}(N)^{-1} = \frac{\sinh(\lambda_1/2) \sinh(\lambda_2/2)}{\lambda_1 \lambda_2 / 4}, \quad (159)$$

so that using again (146), the anomaly polynomial reads

$$I_8 = \hat{A}(R) \left[ K \left( \text{tre}^{iF} + \frac{\lambda_1 \lambda_2}{2} K \right) - \sum 2^{i+1} K_{2i} \right], \quad (160)$$

where

$$K = \frac{\sinh(\lambda_1/2) \sinh(\lambda_2/2)}{\lambda_1 \lambda_2 / 4} \text{tre}^{iG}. \quad (161)$$

Equation (160) can be cast in the form

$$\begin{aligned} I_8 &= MX_8 + K_2 X_6 + \left( K_4 - \frac{M}{48} p_1(R) \right) X_4 + K_6 X_2 \\ &+ \lambda_1 \lambda_2 \left( -\frac{M^2}{48} p_1(R) + MK_4 + \frac{1}{2} K_2^2 \right). \end{aligned} \quad (162)$$

We easily deduce the  $Y$  polynomials, given by

$$\begin{aligned} Y_2 &= K_2 = i \text{tr} G, \quad Y_4 = K_4 - \frac{M}{48} p_1(R) = -\frac{1}{2} \text{tr} G^2 + \frac{M}{24} p_1(N) + \frac{M}{96} \text{tr} R^2, \\ Y_6 &= K_6 = -\frac{i}{6} \text{tr} G^3 + \frac{i}{24} \text{tr} G p_1(N). \end{aligned} \quad (163)$$

The last line in (162) can be cast in the form

$$\chi(N) \left( MY_4 + \frac{1}{2} Y_2^2 \right), \quad (164)$$

which completes the derivation of (92).

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